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RESEARCH NOTE

Constraint Analysis and Initial Sizing of a Transport Aircraft by Simulated Annealing

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This paper discusses a novel approach to the initial sizing process in aircraft conceptual design. In the classical approach, the design point is obtained either graphically or by simultaneous solution of the constraint equations. In this approach, the design point is obtained by coupling the constraint analysis and Sizing procedure to an optimization algorithm, and then arriving at the optimum value of an objective function, subject to the constraints. As an example, the initial sizing of a jet-engined transport aircraft is carried out by coupling Loftin's constraint analysis and sizing procedure for minimum take-off weight to SIMANN Simulated Annealing optimization algorithm. The wing loading and power loading for minimum take-off weight obtained by this approach compare very well with the values obtained using the graphical approach. The results obtained by applying this method for constraint analysis and sizing for the two other objective functions are also presented.

NOMENCLATURE

AR_W	Wing aspect ratio
$C_{L,App}$	Approach lift coefficient
$C_{L,TO}$	Take-off lift coefficient
D	Drag (N)
L	Lift (N)
M_{cr}	Cruise Mach Number
R	Range (km)
S	Wing area (m^2)
T_{SL}	Thrust at sea level (N)
T	Thrust (N)
U	Useful load fraction
W_{TO}	Take-off gross weight (N)
W_f	Fuel fraction
W_e	Empty weight (N)
W_p	Payload Weight (N)
γ_{SSCG}	Second stage climb gradient
γ_{MAG}	Missed approach gradient

Abbreviations

LFL	Landing Field Length
OEI	One Engine Inoperative
SFC	Specific Fuel Consumption
TOFL	TakeOff Field Length
min	Minimum
max	Maximum

INTRODUCTION

The first step in aircraft conceptual design is the initial sizing, in which certain key parameters of the aircraft such as the Takeoff gross weight (W_{TO}), Thrust at sea-level conditions (T_{SL}) and wing area (S) are estimated, based on certain requirements that the aircraft is expected to meet. The end-user of the aircraft being designed lists specific performance and operational requirements. Some requirements also arise from the Airworthiness regulations, such as the minimum climb gradients during climb and missed-approach stages, which ensure that the aircraft is safe to operate. It can be shown that for a jet-engined transport aircraft, the two most important aircraft parameters that affect the performance capabilities are the Takeoff Thrust Loading (T_{SL}/W_{TO}) and the Takeoff Wing Loading (W_{TO}/S) [1]. Aircraft designers use a method called

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constraint analysis to narrow down the choices and help them focus on the most promising concepts.

Constraint Analysis And Initial Sizing

In constraint analysis, ranges of values for an aircraft concept's W_{TO}/S and T_{SL}/W_{TO} are calculated, which enable the design to meet the specified requirements. Once the values of these two parameters are obtained, initial sizing of an aircraft can be carried out, if W_{TO} can be estimated by carrying out the mission analysis, or using some other operational requirement(s).

In a typical constraint analysis procedure, the requirements are converted into relationships between W_{TO}/S and T_{SL}/W_{TO}, which are then plotted as constraint lines. The solution space lies "inside" all of the boundaries set by the various constraint lines. The point within the design space that corresponds to the optimum value of some objective function is then chosen as the design point, such as the one shown in Figure 1. The method of constraint analysis is described in detail by Raymer [1] and Brendt et al [2].

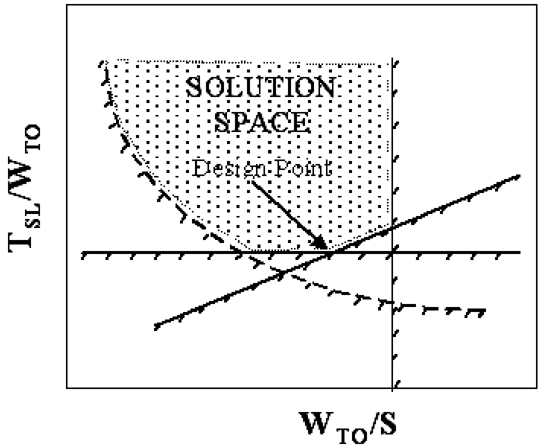

Figure 1. A Typical Constraint Diagram

The design point can be determined graphically by inspection after plotting the constraint diagram. In most cases, the design point lies at the intersection of two or more constraint lines, hence it can also be obtained by determining the point(s) of intersection of all the constraint equations and then choosing the best point(s) that lie(s) in the feasible region. This process is quite cumbersome and unwieldy, and has to be repeated for all objective functions that the designer would like to examine.

This paper proposes a new method for quickly arriving at the design point and carrying out the initial sizing of a transport aircraft. The method involves coupling of the constraint analysis and sizing procedure presented by Loftin [3] to the SIMANN Simulated Annealing optimization algorithm [4, 5]. As an example, the design point of transport aircraft is determined for a given set of user-specified airworthy requirements for three different objective functions, and the initial sizing is carried out.

METHODOLOGY FOR CONSTRAINT ANALYSIS AND SIZING

Loftin [3] has described a methodology for a rapid estimation of the size, weight and thrust required from a transport aircraft to meet user-specified requirements. The methodology consists of two distinct modules, the constraint analysis module and the sizing module.

In the constraint analysis module, the main design parameters are Approach Lift Coefficient ($C_{L,App}$), Takeoff Lift Coefficient ($C_{L,TO}$) and the Wing Aspect Ratio (AR_w). The wing loading required to meet the Landing Field Length requirement is estimated using $C_{L,App}$. Utilizing $C_{L,TO}$, a relationship between T_{SL}/W_{TO} and W_{TO}/S to meet the TakeOff Field Length requirement is obtained. By using AR$_w$ and a few other aerodynamic parameters such as lift and drag increments due to flap deflection; the Lift-to-Drag ratio (L/D) of the aircraft during climb, approach and cruise is estimated. Relationships between T_{SL}/W_{TO} and W_{TO}/S to meet the second stage climb gradient (γ_{SSCG}) and missed approach gradient (γ_{MAG}) are then obtained. The maximum L/D under cruising conditions is determined, and assuming some power-plant characteristics, a cruise matching analysis is carried out, resulting in an expression between T_{SL}/W_{TO} and W_{TO}/S.

The sizing module involves determination of W_{TO} using the cruise range requirement. First, the fuel fraction (W_f/W_{TO}) required to meet the Range (and Diversion) requirement is estimated using the Breguet range approximation for gross still air range in constant altitude flight. The useful load fraction of the aircraft U (defined as 1- W_e/W_{TO}, where W_e is the aircraft empty weight) is determined, using a correlation with T_{SL}/W_{TO} for several existing transport aircraft. W_{TO} is obtained using a simple relationship between W_p, U, and W_f/W_{TO}. T_{SL} and S can then be easily determined, which completes the sizing process. Details of Loftin's methodology are available in [3].

Constraint Analysis And Sizing As An Optimization Problem

Loftin's constraint analysis and sizing procedure for minimum W_{TO} as the objective function can be recast as an optimization problem as follows:

Minimize: W_{TO} = f (W_{TO}, S, T_{SL}, AR_W, $C_{L,TO}$, $C_{L,APP}$).

Subject to the Constraints g_1 to g_5, viz.:

$$LFL = g_1(W_{TO}/S, C_{L,App}) \leq LFL_{max}$$

$$TOFL = g_2(W_{TO}/S, T_{SL}/W_{TO}, C_{L,TO}) \leq TOFL_{max}$$

$$\gamma_{SSCG} = g_3(T_{SL}/W_{TO}, C_{L,TO}, AR_w) \geq \gamma_{SSCG,min}$$

$$\gamma_{MAG} = g_4(T_{SL}/W_{TO}, C_{L,App}, AR_w) \geq \gamma_{MAG,min}$$

$$M_{cr} = g_5(T_{SL}/W_{TO}, W_{TO}/S, AR_w) \geq M_{cr,min}$$

It may be noted here that the objective function is implicit in W_{TO} as the objective function is also one of the design variables. This formulation ensures that the lowest value of W_{TO} that meets all the constraints emerges as the solution. Coupling the constraint analysis and sizing procedure with any multi-variable constrained optimization algorithm can solve this problem; thus, obviating the need for graphical solution or simultaneous solution of all the constraint equations. Further, the design point and sizing can also be carried out for other objective functions apart from W_{TO}, (such as minimum W_f/W_{TO} or maximum U), without changing the basic formulation.

AN EXAMPLE OF CONSTRAINT ANALYSIS AND SIZING

As an example, the method was applied for the constraint analysis and initial sizing of a transport aircraft designed for R = 2500 km and W_p =15000 kg, which is expected to meet the performance and operational requirements listed in Table 1.

The variation of Thrust available and SFC, with cruise Mach number for various altitudes for four engines are provided in [3], but they were considered too old to be used in the present study. Instead, the variation of these parameters at $M_{cr} = 0.8$ for CFM56 engine were generated from sea level to 17000 m, using the engine cycle analysis and sizing model developed by Sanghi [6]. A cubic polynomial fit was obtained for the thrust lapse with altitude at this Mach number for CFM 56.

For optimization, the SIMANN code developed by Goffe et al [4], based on the Simulated Annealing algorithm developed by Corana et al [5] was employed. The efficacy of this code in tackling complex objective functions has been demonstrated in a previous study [7]. An exterior penalty function approach was employed to handle the constraints, as they cannot directly be handled in SIMANN. Details of the SIMANN code are not provided here due to paucity of space.

A constraint analysis was carried out using the Loftin's method described above, with the modified power plant model. The results of the constraint analysis are shown graphically in Figure 2, for $AR_W = 9.5$, $C_{L,TO} = 2.0$ and $C_{L,APP} = 2.8$, for the objective function of minimum W_{TO}. It can be seen that the design drivers in this case are TOFL and M_{Cr}, and the design point corresponds approximately to $W_{TO}/\mathrm{S} = 6610\ \mathrm{N/m^2}$ and $T_{SL}/W_{TO} = 0.3840$.

A comparison between the salient parameters obtained by the graphical method, as well as the new methodology for the initial sizing for minimum W_{TO} is shown in Table 2. It can be seen that most of the parameters obtained by both methodologies are very similar. The only constraints that are active at the design point are those on TOFL and M_{cr}.

CONSTRAINT ANALYSIS AND SIZING FOR OTHER OBJECTIVE FUNCTIONS

The new methodology was applied for constraint analysis and sizing for the two other objective functions viz. minimum W_f/W_{TO} (Case-2) and maximum U

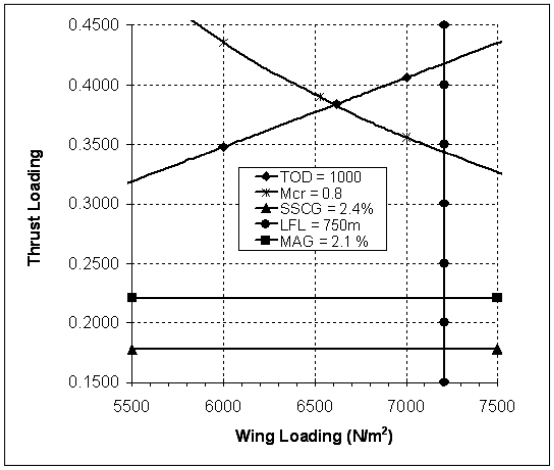

Figure 2. Constraint Diagram for minimum W_{TO}

Table 1. Requirements for constraint analysis and sizing

Parameter	Performance and Operational Requirements
TOFL	1000 m at airfield altitude of 3000 m, Temperature = 25^0 C, with W_p = 6 tons & R =1500 km
LFL	750 m at airfield altitude of 3000 m, Temperature = 25^0 C, at Max. Landing Weight
γ_{MAG}	2.1 %, at ambient temperature = 25^0 C, at Max. Landing Weight, OEI
γ_{SSCG}	2.4% , at Maximum W_{TO}, at ambient temperature = 25^0 C with OEI
M_{cr}	0.8, at ISA+15^o C

Table 2. Salient results of the sizing study for minimum W_{TO}

Parameter	Units	Graphical Method	New Method
DESIGN POINT			
W_{TO}/S	N/m^2	6610	6618.4
T_{SL}/W_{TO}	-	0.3840	0.3840
DESIGN VARIABLES			
W_{TO}	N	568129	568171
A_{RW}	-	9.500	9.495
T_{SL}	N	109082	109076
$C_{L,TO}$	-	2.00	1.971
S	m^2	85.90	85.85
$C_{L,APP}$	-	2.8	2.8
CONSTRAINTS			
	-	Calculated Value	Desired value
LFL	M	688.6	$\leq$750
TOFL	M	1000	$\leq$1000
γ_{SSCG}	%	14.20	$\geq$2.4
γ_{MAG}	%	12.27	$\geq$2.1
M_{cr}	-	0.80	= 0.80

Table 3. Salient results of the sizing study for the three objective functions

Parameter	Units	Min. W_{TO}	Min. W_f/W_{TO}	Max. U
W_{TO}/S		6618.2	6774.4	6465.6
T_{SL}/W_{TO}		0.3840	0.3938	0.3753
W_{TO}	N	568171	585922	586753
A_{RW}		9.495	9.494	7.027
T_{SL}	N	109076	115364	110089
$C_{L,TO}$		1.971	1.975	1.874
S	m^2	85.85	86.49	90.75
$C_{L,APP}$		2.8	2.8	2.8
W_f/W_{TO}		0.1099	0.1075	0.1271
U		0.3688	0.3586	0.3779
LFL	m	688.7	705.3	672.9
TOFL	m	1000	998.6	999.9
γ_{SSCG}	%	14.42	14.63	11.79
γ_{MAG}	%	12.27	12.88	8.94
M_{cr}	-	0.80	0.82	0.8
W_e/W_{TO}	-	0.6312	0.6414	0.6221
W_e	N	358630	375810	365019
W_f	N	62442	62987	74576
W_p/W_{TO}	-	0.2589	0.2511	0.2508

(Case 3). The values of some important parameters for these cases are summarized in Table 3, along with those for minimum W_{TO} (Case-1). It may be noted that in all the three cases, the constraint on the TOFL is active, and value of $C_{L,APP}$ is always on the upper limit to ensure that the constraint on LFL is met. In Case-1, lowest W_{TO} is achieved by using the smallest power plant (lowest (T_{SL}) and wing (lowest S) that meets the requirements and constraints; thus, resulting in the lowest W_e and W_f. Since W_p is the same for all cases, this case also corresponds to the highest payload fraction W_p/W_{TO}.

In Case-2 the objective of lowest W_f/W_{TO} results in the highest values of W_{TO}/S and T_{SL}/W_{TO}, which in turn lead to the largest value of T_{SL} and W_e (and also W_e/W_{TO}). Highest T_{SL}/W_{TO} also results in large γ_{SSCG} and γ_{MAG}, and the highest W_{TO}/S results in the largest LFL.

In Case-3, on the other hand, W_{TO}/S and T_{SL}/W_{TO} are the lowest, while W_{TO}, S and W_f (and also W_f/W_{TO}) are the highest. This is because in Loftin's methodology, U is directly related only to T_{SL}/W_{TO}; thus, U is maximized only by minimizing T_{SL}/W_{TO}. The lowest value of AR_w and highest S result in the highest induced drag (hence fuel consumption), leading to the highest W_f. The low value of T_{SL}/W_{TO} results in the least γ_{SSCG} and γ_{MAG}, though much above the minimum required values. This case also corresponds to the lowest value of W_p/W_{TO}.

CONCLUSIONS

The above results indicate that the new method for constraint analysis and sizing is a useful alternative to the classical approach, which involves a graphical solution or simultaneous solution of constraint equations for each objective function. An advantage of this method is that a closed form mathematical formulation of the objective function or the constraints with respect to the design variables is not required. This is very useful in aircraft conceptual design, since many a times, the constraints have to be evaluated by an iterative procedure, as the relationships are in the form of implicit equations. While Loftin's methodology provides the expressions for only the above-mentioned five constraints, it is also possible to incorporate other constraints in this method as long as they can be evaluated in some way as functions of the design variables.

The results also indicate that the formulation of Loftin's methodology needs to be modified, in order for it to lead to meaningful results. For instance, in Case-3, maximization of the useful load fraction U results in the highest value of W_{TO}, and W_f/W_{TO}, which is absurd.

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