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RESEARCH NOTE

On Efficiency of an Unstructured Grid Generation Method for Aerospace Applications

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A procedure is presented for efficient generation of high-quality inviscid and viscous unstructured grids applicable to aerospace flow simulations. The method uses an iterative point creation and insertion scheme. Points are created using advancing-front/layer type point placement and the connectivity for these generated points are obtained by directly subdividing the elements, which contain them without regard to the quality. This connectivity is then improved by iterative local reconnection subject to a quality criterion. Different point placement schemes are used together with two quality criterions, Delaunay in-circle and min-max, the latter generating better quality grids. Results showed that special care must be taken in the generation of initial grid, particularly when the distribution of spacing is based on the spacing of the boundary edges. Computational results are also presented for a laminar flow around hyperbola nose using viscous grid generated with the present method.

INTRODUCTION

Computational fluid dynamics (CFD) has become an efficient tool for simulation of complex flows around aerospace configurations. In order to handle both complex geometry and physics unstructured grid technology is commonly used in CFD simulations [1]. Several unstructured grid generation procedures have been developed, typically based on either an advancing-front [2,3] or Delaunay [4,5] approach. The advancing-front approach offers advantages of high-quality elements and integrity of the boundary while it suffers from efficiency factors. The Delaunay method, in contrast, offers high efficiency and a known mathematical basis, but the quality of elements is not guaranteed in some cases (i.e. high aspect ratio viscous grids).

Thus, none of these procedures have combined characteristics of efficiency, quality and robustness. Recent research has therefore focused on the development of methods in order to provide improved overall characteristics. Methods using a combined approach with advancing-front type point placement and Delaunay connectivity have been developed for two-dimensional geometries [6,7,8,9]. These methods are similar to a typically Delaunay approach in that they start with a valid triangulation which includes all boundary points. New points are then generated using an advancing-front type placement. A Delaunay criterion is used for the insertion of new points into the existing triangulation. The method proposed by Marcum and Weatherill [10] utilizes edge-swapping algorithm of Lawson to locally satisfy the min-max criterion, which minimizes the maximum angle of triangles. In some cases, this method may not lead to the most suitable grid. Thus, further improvements are needed.

The main objective of the present work is to investigate the efficiency of different combinations of techniques in order to achieve an efficient unstructured grid generation procedure for aerospace applications. Different combinations are tested for initial grid generation, point placement and optimum connectivity procedures. Details of the proposed method are presented in the following sections for isotropic and high aspect ratio viscous grids. Finally, in order to demonstrate the capability of the method for calculation of applied aerospace problems, numerical solution of the flow around a hyperbola nose at high-speed conditions is presented.

ISOTROPIC GRID GENERATION PROCEDURE

Isotropic (equilateral) grids are generally suitable for inviscid part of the flow domain, outside boundary layer

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and wakes, where no preferred direction for stretching is recognized. The proposed method is essentially a combination of automatic point creation, advancing-front point placement, and connectivity optimization schemes. A valid grid is maintained throughout the grid generation process, which allows the grid to serve as an efficient means of searching with a simple data structure. It also provides an efficient way to smoothly distribute the point spacing within the flow domain. Grid point distribution is controlled by a point distribution function, which is either interpolated from initial boundary point spacing or calculated from a distribution of pre-specified line and point sources. An automatic point creation scheme is used, in which points are generated using advancing-front type placement. Connectivity for these generated points is initially obtained by directly subdividing the elements which contain them, without regard to element quality. This connectivity is then improved by iteratively using local reconnection subject to a quality criterion. The overall procedure is applied repetitively until a complete field grid is generated with a desired point distribution.

The necessary steps in the basic procedure are outlined as follows:

1. Generate a boundary (surface) grid for the given configuration.
2. Obtain a valid triangulation of the boundary points and recover all boundary surfaces.
3. Assign a point distribution function to each initial boundary point by averaging the length of edges connecting to that point.
4. Initialize the data structure. For each element, the element edges and for each edge, the edge points and neighboring cells, and an active/off flag are saved. Initially all elements are made active.
5. Turn off each active element, which satisfies the point distribution function.
6. Create a new point for each active element.
7. Interpolate the point distribution function for the new point from the containing element.
8. Reject new points that are too close to an existing point or another new point.
9. Subdivide the containing element. Make all new elements active.
10. For each active element, compare the reconnection criterion for all allowable connectivities with adjacent elements and reconnect using the most optimal connectivity.
11. Repeat the local reconnection process, step10, until no elements are reconnected.
12. Repeat the field point generation process, steps 5-11, until no new points are created.
13. Smooth the coordinates of the field grid points.
14. Repeat the iterative local reconnection, steps 10 and 11, with all elements activated.

Figures 1(a-f) show steps of the above unstructured grid generation procedure. For this case, the initial grid contains 124 elements and 62 boundary points and the final grid contains 939 elements. In most cases the initial grid inevitably composed of a significant number of high-aspect ratio elements. However, the overall procedure appears to converge to a high-quality grid. In order to demonstrate the capability of the method to generate grids around complex geometries, the final grid generated by this method around a modern car is shown in Figure 2a. The grid contains 320 boundary points and 5771 elements. An element angle distribution with 5-deg increment is shown in Figure 2b, which illustrates the concentration of element angle distribution around the optimum value of 60-deg.

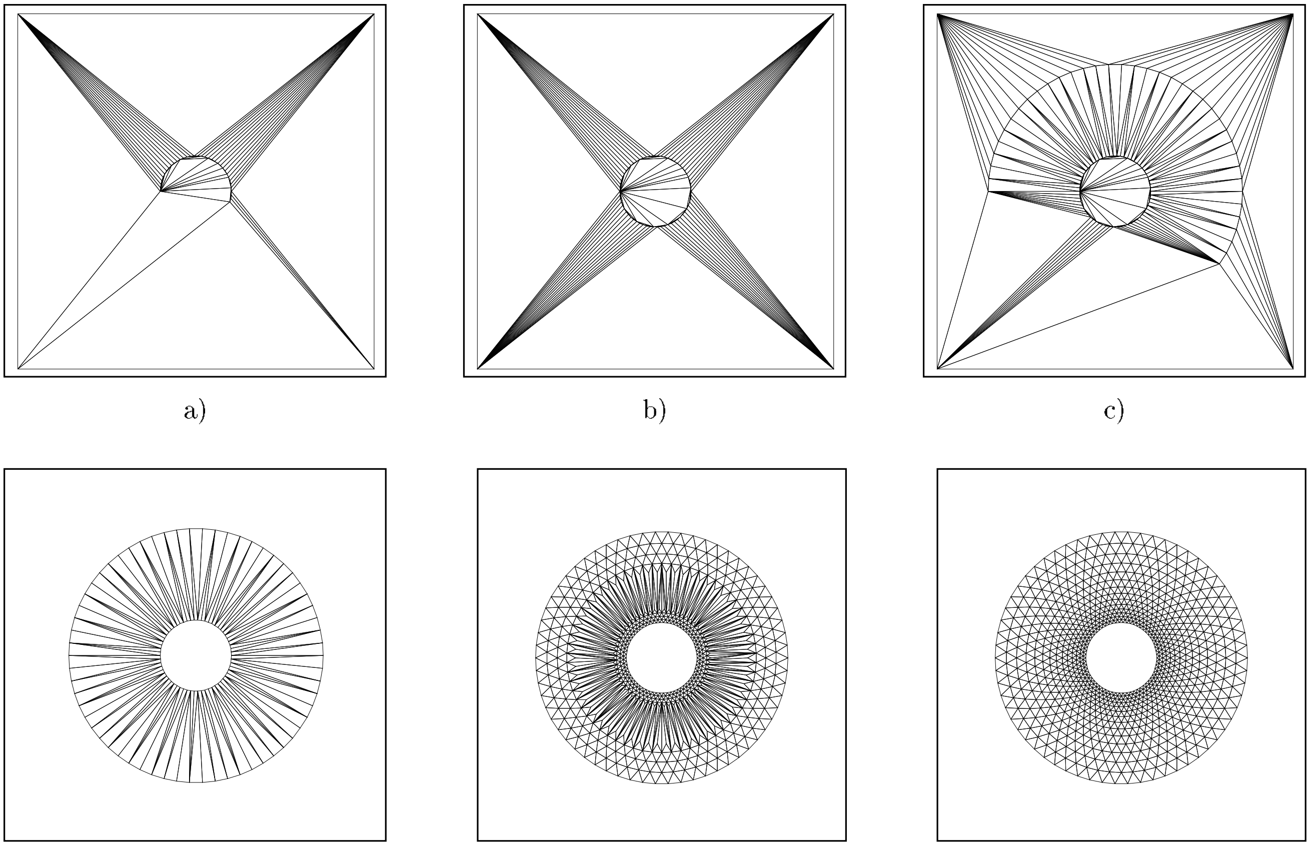
VISCOUS GRID GENERATION PROCEDURE

Applied aerodynamic problems usually concerned with high Reynolds number flows with very thin boundary layer. Accurate solution of such flows in which high velocity gradients are present in (surface) normal direction requires generation of non-isotropic grids. The previous steps for isotropic grid generation, thus, requires some modifications in the point placement strategy in order to be used for high aspect ratio grid generation. The advancing-normal point placement method is used in this work [12]. Points are generated by advancing along the normal line on the surface boundary points with a proper stretching ratio. The main advantage of this method is the possibility of extending the method easily to three-dimensions. The necessary steps in this procedure are outlined below.

1. Determine the outward normal unit vectors for each face (boundary edges).
2. Determine the boundary point normals that are the average of the two normals of the neighboring edges.
3. Since rapid changes in slopes of the normals specially on the corners, cause outward normal unit vectors crossover each other, they should be smoothed iteratively by a Laplacian smoothing operation. The iterative formulation is,

$$v_p^{t+1} = (1-\omega)v_p^t + \frac{\omega}{N_p}\sum_{n=1}^{N_p} v_n^t \qquad (1)$$

where v_p is the normal unit vector, t is an iteration level, ω is a relaxation coefficient (normally between



a) b) c)

d) e) f)

Figure 1. Initial, intermadiate and final grids for cocentric circles

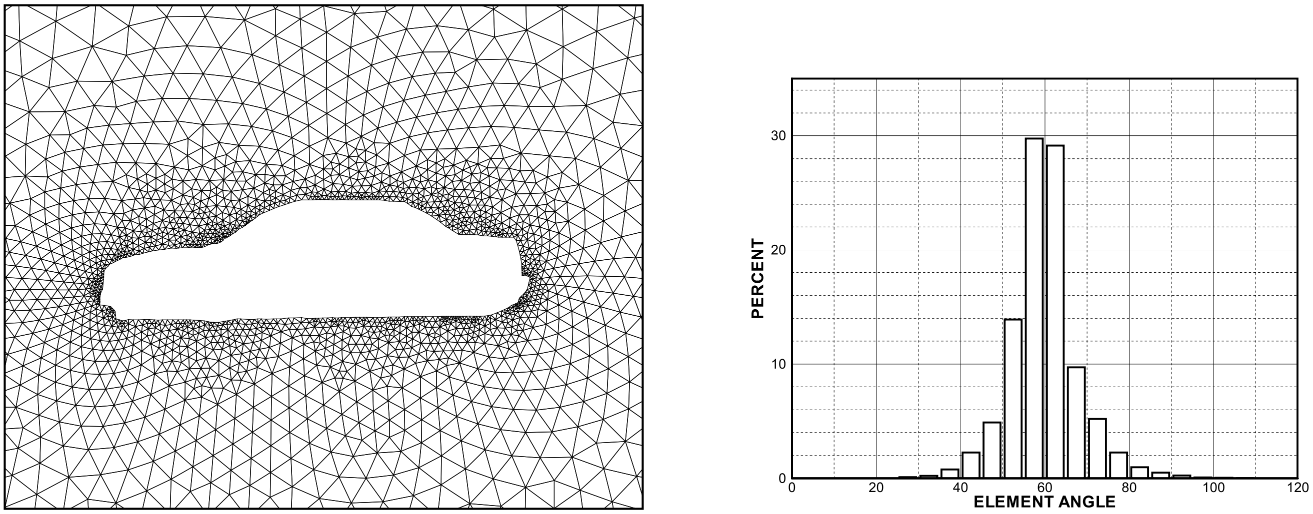
0 and 2), N_p is the number of the neighbors of the point P and v_n is the neighbors normal unit vector.

4. Create a new point at a position along each normal.

New points are generated by;

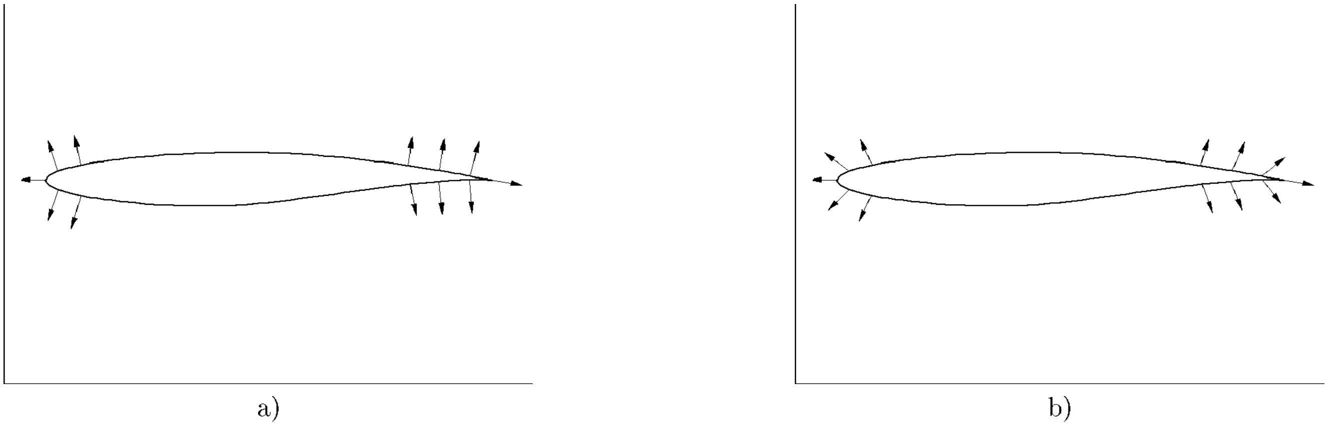
$$X_p^{n+1} = X_p^n + v_p \delta \eta^{n-1} \tag{2}$$

where X_p is the point's coordinate, n is the number



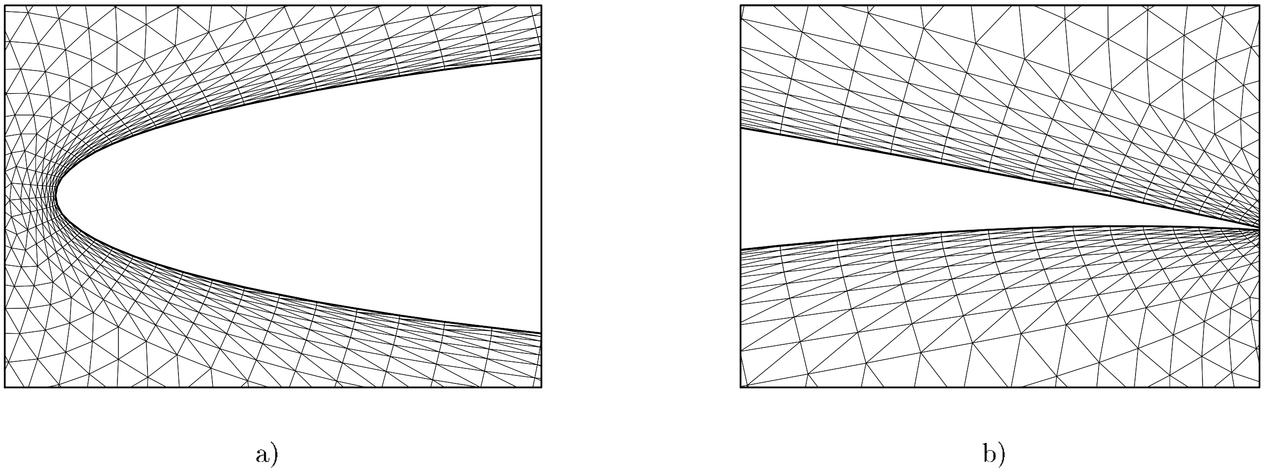
a) b)

Figure 2. a) Final grid around a modern car, b) Element angle distribution



a) b)

Figure 3. Normal unit vectors a) before and, b) after smoothing



a) b)

Figure 4. Effect of smoothing of the normal unit vectors on the viscous layers

of layer, δ is the distance of the first layer to the surface and η is the stretching ratio. For turbulent viscous flow over an airfoil $\delta = 10^{-4}$ and $\eta = 1.2$ are recommended.

5. In order to avoid the creation of apex angles around the convex corners such as trailing edge, the relaxation coefficient should be renewed at each level of advancing by,

$$\omega = \min(1, \frac{n-1}{\alpha N}) \tag{3}$$

where N is the final number of layers and α is chosen less than one.

6. Repeat steps 1-5 to achieve the desired number of layers After completing the generation of high aspect ratio grids inside shear layer the steps mentioned in the previous section is followed for isotropic triangular grid generation outside viscous layer.

Figure 3 shows how the smoothing process can decrease the rapid changes in slopes near discountinuities. This effect can be seen in Figure 4 near the leading and trailing edges of the RAE 2822 airfoil. The final viscous grid around this airfoil is shown in Figure 5.

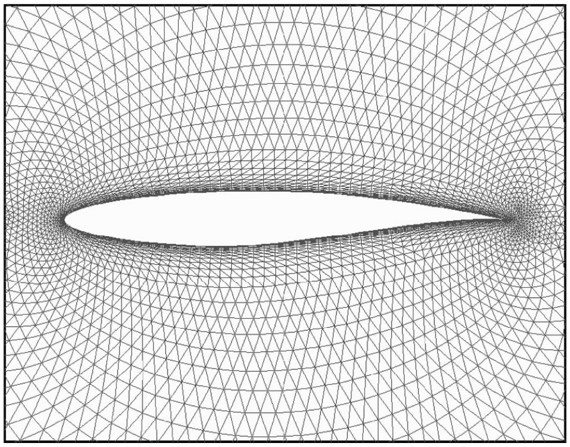


Figure 5. Generated grid around RAE 2822 airfoil

GRID QUALITY ENHANCEMENT

Several investigations have been made in the present work in order to find the best combination of schemes for point creation, point placement and optimum connectivity.

Initial Grid

The first attempt was regarding to the initial grid. When using the spacing of the boundary points for point distribution in the domain, a desirable variation was found between the fine spacing on the interior boundaries and the coarse spacing on a far field boundary. But along some complex boundaries, it may happen that points are connected between finely spaced interior boundaries leading to non-smooth grid cells near these areas. This behavior is clearly illustrated in Figures 6a,b where initial and final generated grids around a 4-elemet landing configuration are shown. After some investigation, it was found that by applying extra min-max reconnection after final stage of the initial grid generation this problem would vanish. Figures 7a,b show the effect of the proposed method to enhance the quality of the grid.

Point Distribution Function

Another research was focused on alternative point distribution functions. Following Jahangirian and Johnston [11] point and line sources are introduced within the domain and for each cell j, the spacing d_{sj} is given by:

$$d_{sj} = (r_{j1} + r_{j2} - D_{12})B \tag{4}$$

where r_{j1} and r_{j2} are the distances from the cell center to the edges of the line source, and D_{12} is the length of the line source. B is a user-specified constant defining the intensity of the line source and generally takes a

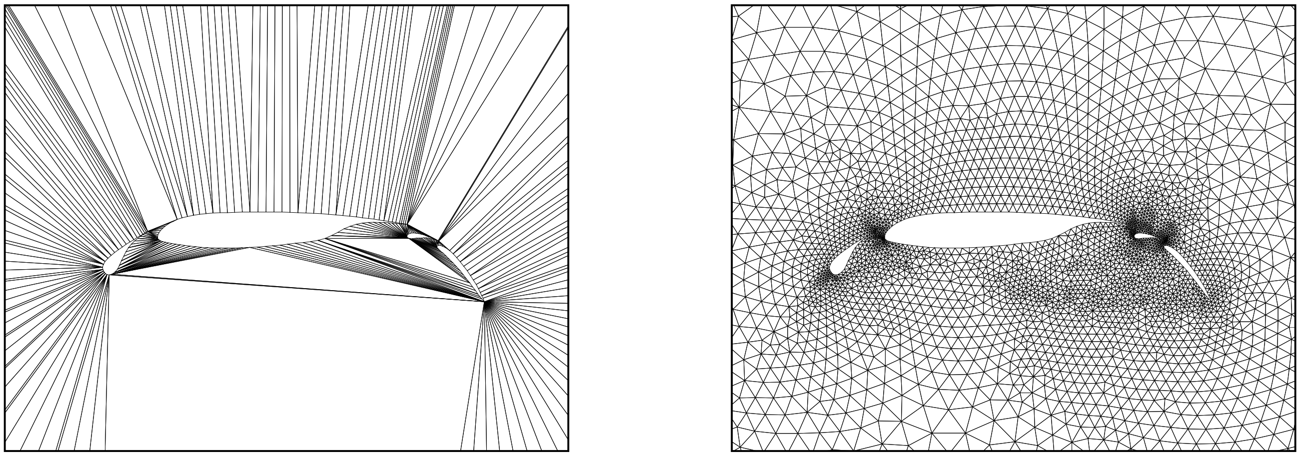

a) b)

Figure 6. a) Initial grid without swapping, b) Final grid

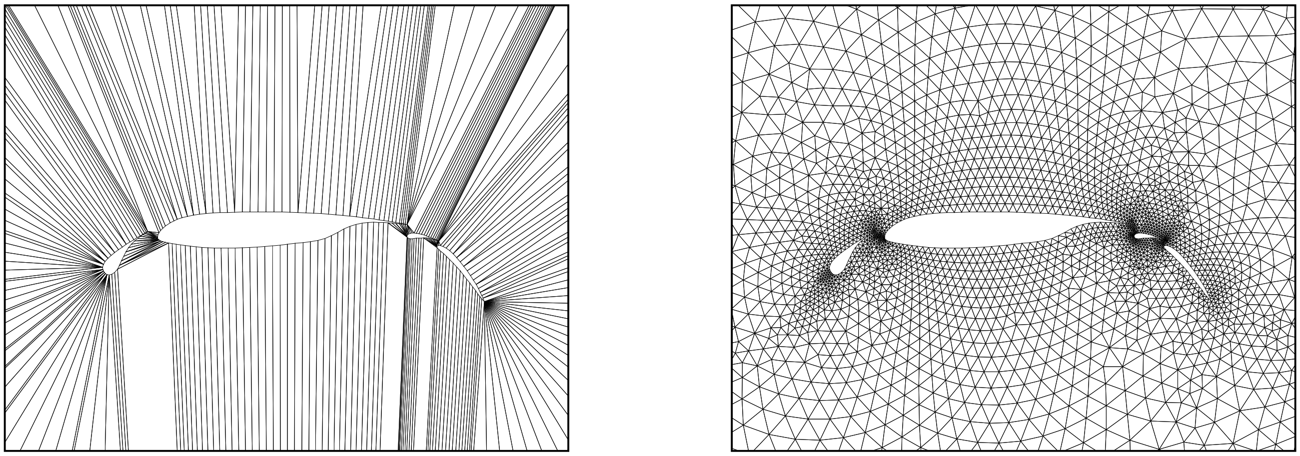

a) b)

Figure 7. a) Initial grid after min-max reconnection, b) Final grid

value between 0.3-1.0. For a point source, r_{j2} and D_{12} are set to zero. When several sources are introduced in the domain, the lowest induced spacing is used for each cell. Figure 9 shows the final grid generated using line and point sources while Figure 8 illustrates the grid generated using the base method. It is clear that both methods can produce grids with similar quality and smoothness but the new method is capable of generating grids with point concentration around posteriori flow features.

Point Placement Strategy

The next investigation was about point placement strategy, which has much influence on the quality of the final grid. Marcum and Weatherill [10] showed that centroid placement leads to the lower quality grid compared to the advancing front point placement strategies. Two different advancing front point placement strategies are studied including the method of peraire et al. [2] and the method of Muller et al. [7]. Figure 10 shows the grid generated based on reference [2] while Figure 8 illustrates the grid generated using the Muller approach. It is clear that better quality is achieved by the method of Muller. This is reflected in the element angle distributions shown in Figure 12.

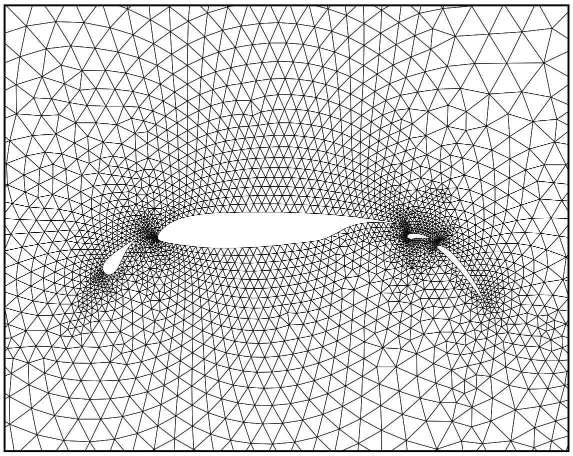


Figure 8. Final reference grid

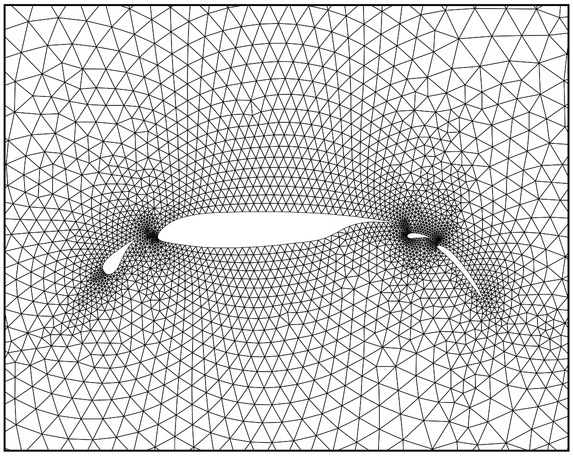


Figure 9. Final grid using line and point sources

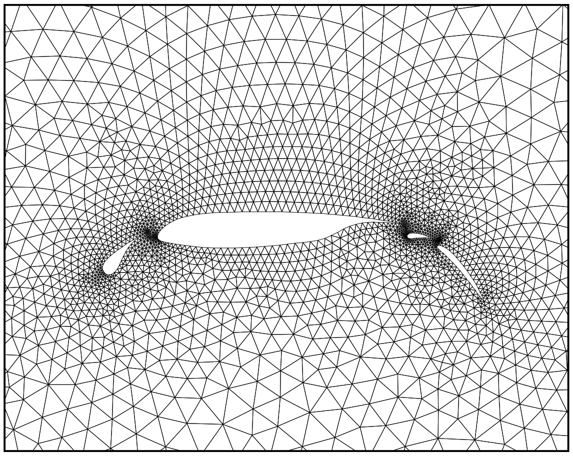


Figure 10. Final grid using Peraire et al. [2] point placement

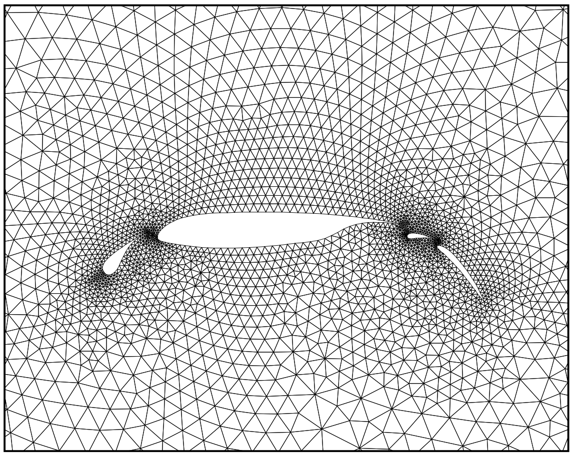


Figure 11. Final grid using Delaunay in-circle criterion

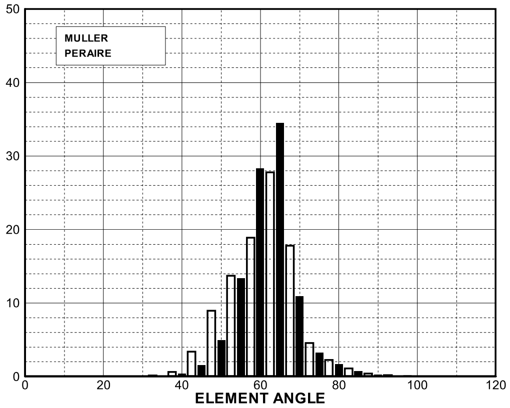


Figure 12. Element angle distributions

Optimum Connectivity Procedure

The final investigation was to find the optimum connectivity procedure. A local reconnection scheme based on the edge swapping algorithm of Lawson is used. In this approach, the grid is repetetively reconnected or swapped to locally satisfy a desired quality criterion. The process is repeated until no new reconnections are possible. Two quality criterions, Delaunay in-circle and min-max, are considered and the generated grids are compared. Figure 11 shows the final grid generated using the Delaunay in-circle criterion while Figure 8 illustrates the final grid with min-max criterion. It may be recognized that the min-max criterion which minimize the maximum angle of the elements provide improved smoothness; however, the element angle distributions shows almost no difference as illustrated in Figure 13.

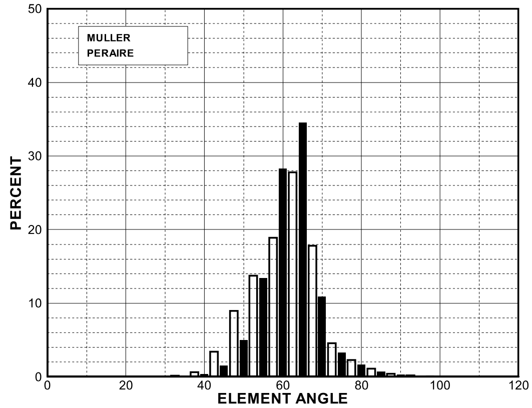
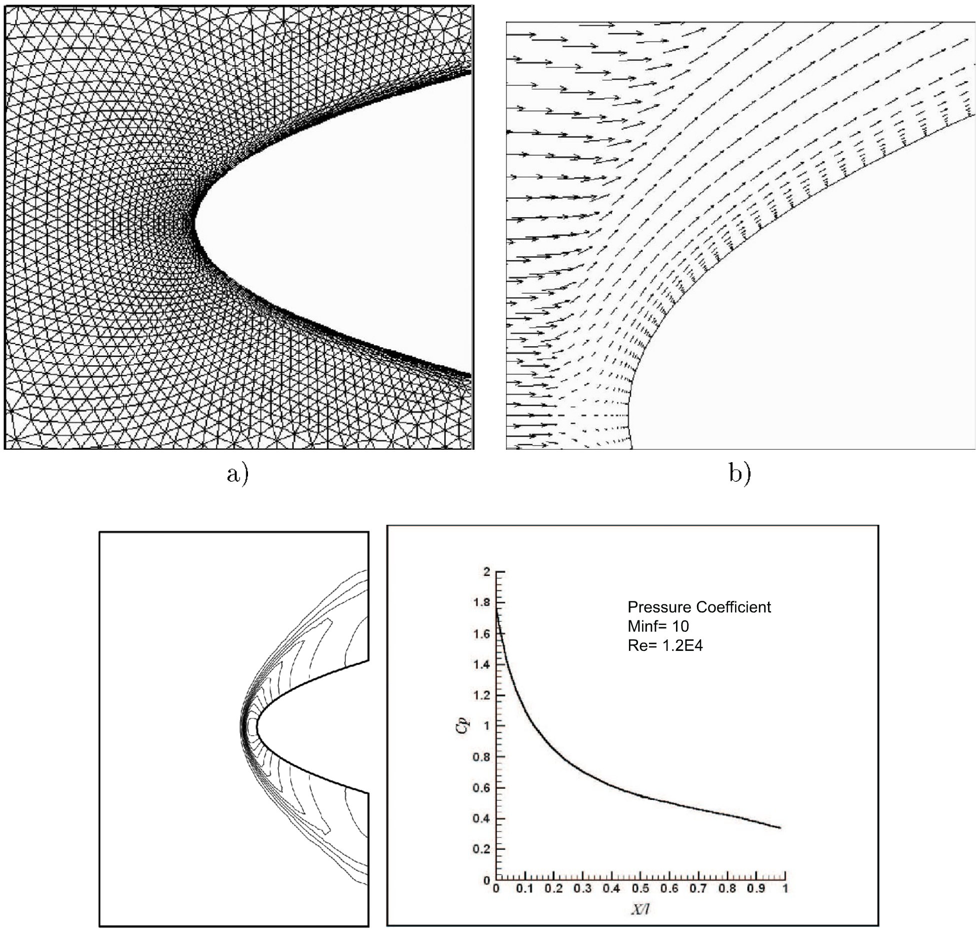


Figure 13. Element angle distributions

Flow Solution Example

A numerical flow simulation is presented in order to demonstrate the capability of the method to generate the quality grids for applied aerospace problems. A high-speed laminar flow around the hyperbola geometry with mach number 10 and Reynolds number 12000 is considered. A finite-volume cell-centered explicit scheme is used for solution of Navier-Stokes equations. The details of the flow solver can be found in reference [1]. The generated viscous grid is shown in Figure 14a. Figure 14b illustrates velocity vectors



a) b)

c) d)

Figure 14. a) Generated grid b) velocity vectors, c) pressure contours and d)Cp around a hyperbola geometry, $M_{inf} = 10$ and Re=12000.

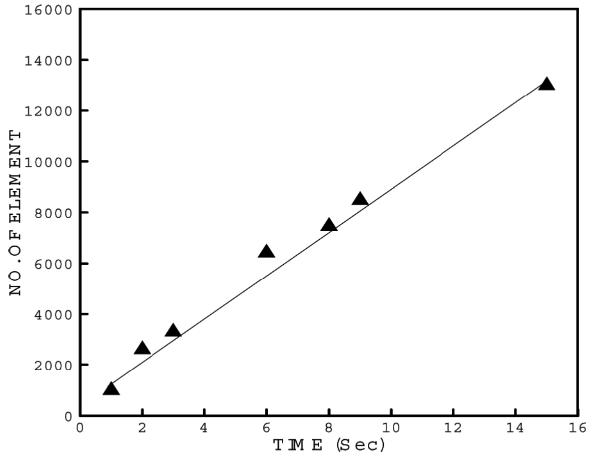


Figure 15. CPU time required by pentium II 333

within the domain where the velocity profiles inside boundary layer are well captured with the present grid. The pressure contours are shown in Figure 14c. The comparison of the surface pressure coefficient distributions with the numerical data presented in reference [13] is shown in Figure 14d. Good agreements are achieved as illustrated in this figure. The unstructured grid generation procedure described in the preceding sections has been implemented in a computer code written in Fortran. Required CPU times for a variety of configurations on a Pentium II 333 MHz are shown in Figure 15 where linear variation of CPU times with the number of generated cells is apparent. This behavior usually demonstrates the efficiency of the method in terms of computations especially for search routines.

CONCLUSIONS

A procedure has been presented for efficient generation of high-quality inviscid and viscous unstructured grids. The present procedure is based on an iterative point insertion scheme using advancing-front type placement and local reconnection. Several combinations of methods have been tested, and it was found that the best quality would be achieved with Muller et al. point placement strategy together with min-max reconnection quality criterion. A method is presented for enhancement of initial grid quality. Suitable distribution of grid spacing is also obtained using point and line sources. Numerical solution of a laminar viscous flow around a hyperbola nose at mach 10 was presented and comparisons with the reference data showed good agreements

REFERENCES

1. Jameson, A., Baker, T.J. and Weatherill, N.P., "Calculation of Inviscid Transonic Flow Over a Complete Aircraft", *AIAA Paper*, 86-0103, (1986).
2. Peraire, J., Vahdati, M., Morgan, K and Zienkiewicz O.C., "Adaptive Remeshing for Compressible Flow Computations", *Journal of Computational Physics*, **72**, (1987).
3. Lohner, R. and Parikh, P., "Three-dimensional Grid Generation by the Advancing Front Method", *Int. Journal of Numerical Methods in Fluids*, **8**, PP 1135-1149(1988).
4. Weatherill, N. P., "A Method for Generating Irregular Computational Grids in Multiply Connected Plannar Domains", *Int. Journal of Numerical Methods in Fluids*, **8**, PP 181-197(1988).
5. Baker, T.J., "Three Dimensional Mesh Generation by Triangulation of Arbitrary Point Sets", *AIAA Paper*, 87-1124, (1987).
6. Mavriplis, D.J., "An Advancing Front Delaunay Triangulation Algorithm Designed for Robustness", *AIAA Paper*, 93-0671, (1993).
7. Muller, J.D., Roe, P.L. and Deconinck, H., "A Frontal Approach for Internal Node Generation in Delaunay Triangulation", *Int. Journal of Numerical Methods in Fluids*, **17**, PP 241-256(1993).
8. Rebay, S., "Efficient Unstructured Mesh Generation by Means of Delaunay Triangulation and Bowyer-Watson Algorithm", *Journal of Computational Physics*, **106**, PP 125-138(1993).
9. Mazaheri, K. and Jadidi, M., "Unstructured Grid Generation by a Combination of Delaunay Triangulation and Advancing Front Method (in Farsi)", Third Fluid Dynamics Conference, Tehran, Iran, (Jan. 1995).
10. Marcum, D.L. and Weatherill, N.P., "Unstructured Grid Generation Using Iterative Point Insertion and Local Reconnection", *AIAA J.*, **33(9)**, PP 1619-1625(1995).
11. ahangirian, A. and Johnston, L.J., "Automatic Generation of Adaptive Unstructured Grids for Viscous Flow Applications", 5th International Conference on Numerical Grid Generation in CFD, Mississippi State University, (1996).
12. Marchant, M., and Weathrill, N., "Unstructured Grid Generation for Viscous Flow Simulations", Proceedings of the 4th International Conference on Numerical Grid Generation in CFD and Related Fields, Swansea, U.K., (1994).
13. Drikakis, D. and Tsangaris, T., "On the Accuracy and Efficiency of CFD Methods in Real Gas Hypersonics", *International Journal for Numerical Methods in Fluids*, **16**, PP 759-775(1993).