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Material Characterization of Structures Using Genetic Algorithm and Viberational Data

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The aim of this work is to present a technique to identify elastic parameters of composite materials. The identification is based on the adjustment of coefficients in an optimization process in which the objective function is defined by the difference between the analytical natural frequencies and the measured ones. Such analytical natural frequencies are obtained by the finite element method (FEM) while the experimental ones are determined by ordinary modal tests. The proposed approach uses Genetic Algorithm (GA) to solve the optimization problem. A few examples are presented to identify the indirect (Non-Destructive) prediction of mechanical properties of homogeneous isotropic, orthotropic, and anisotropic plates for showing the ability, capability, reliability, modeling and performance of this technique.

INTRODUCTION

Over the last few decades, laminated composites have found usage in aerospace, automotive, marine, civil and sport equipment applications. This popularity is due to excellent mechanical properties of composites as well as amenability to tailoring those properties. Many researchers have continuously devoted their efforts to predict mechanical properties of laminated composite structures by making various assumptions. Although the properties of certain types of composites, modeled as homogeneous, linear and anisotropic, can be assessed analytically from the elastic properties of its constituents, considerable uncertainty remains due to the approximations involved in the derivation of the relevant formulae as well as the effect of the manufacturing processes to geometrical arrangement and properties of its two phases. Recently, the non-destructive evaluation of elastic properties of composite structures based on the modal test data is more commonly used. This is an inverse problem with the parameters in the mathematical model being adjusted repeatedly until its analytical responses match satisfactorily with those associated with the physical structure. Zelenev et al. [1] presented a unique method for determining the elastic constants of an isotropic plate. The determination of elastic moduli of metallic and compound panels utilizing natural modes of vibration was first carried out by Carne et al. [2]. Wearing et al. [3] employed natural mode results to determine the four elastic constants of an especially orthotropic plate. DeWilde et al. [4, 5] have used a technique to determine elastic properties of anisotropic plates. DeWilde et al. [6] used Kirchhoff assumptions and Bayesian parameter estimation method to determine the linear elastic characteristics of a composite sample through its experimentally measured natural frequencies. Deobald et al. [7] used natural frequencies measured by an impulse technique to determine the Young's modulus, in-plane shear modulus, and Poisson's ratio for each plate. Sol et al. [8] presented a method, which determines the elastic properties of a composite material plate using experimentally measured resonant frequencies. Only thin plates, subjected to small lateral deflections were considered as used in Love-Kirchhoff model. Pedersen et al. [9] presented an indirect identification technique to predict the mechanical properties of composite plate specimens. This technique makes use of experimental eigen frequencies, the corresponding numerical eigenvalue evaluation, sensitivity analysis and optimization. Lai et al. [10] extended the method of Deobald et al. [7] to the study of a generally orthotropic Glass/Epoxy plate. Employing the idea of sensitivity, a more stable iteration scheme was put forward by Lai et al. [11] to determine the properties of two generally orthotropic

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laminates. Lai et al. Araujo et al. [12] proposed a numerical/experimental method for the identification of material parameters of the composite materials. More recently, Mariana et al. [13] presented a technique to identify elastic parameters of composite materials using GA to solve the optimization problem. They used a FEM solution base on a triangular elements with three degrees of freedom per node. In this paper, an indirect identification technique is presented to predict the elastic properties of composite plate specimens. The proposed method combines experimental works for the determination of eigen frequencies of a composite plate along with a finite element method for determination of the corresponding analytical resonant frequencies. The natural frequencies of the composite plates are experimentally obtained using an FFT analyzer. The corresponding analytical eigen frequencies are computed using the finite element method. The identification procedure makes use of a search method based on a genetic algorithm technique starting with a set of initial approximation of the material elastic constants.

GENETIC ALGORITHM (GA)

Genetic Algorithm is an unorthodox search or optimization algorithm, which was first suggested by John Holland in his book Adaptation and Artificial Systems [14]. As the name suggests, the processes observed in natural evolution inspired the Genetic Algorithm. It attempts to mimic these processes and utilize them in solving a wide range of optimization problems. In general, Genetic Algorithm performs directed random searches through a given set of criteria. These criteria are required to be expressed in terms of an objective function, which is usually referred to as a fitness function.

The Genetic Algorithm method requires that the set of alternatives to be searched be finite. To apply them to an optimization problem where this requirement is not satisfied, the set involved must be discretized and the appropriate finite subset must be selected. It is further more required that the alternatives be coded in the chromosomes of some specific finite length, which consist of symbols from a certain finite alphabet. These are called chromosomes; the symbols that form them are called genes, and their set is called a gene pool. The Genetic Algorithm method searches for the best alternative (in the sense of a given fitness function) through chromosome evolution. First, an initial population of chromosomes is randomly selected. Then each of the chromosomes in the population is evaluated in terms of its fitness (expressed by the fitness function). Next, a new population of chromosomes is selected from the given population by giving a greater chance to select chromosomes with higher fitness. This is called the reproduction operation. The new population may contain duplicates. If given stopping criteria (e.g., no change in the old and new population, specified computing time, etc.) are not met, some specific, genetic-like operations are performed on chromosomes of the new population. These operations produce new chromosomes, called offsprings. The same steps,i.e., evaluation and reproduction operation are then applied to chromosomes of the resulting population. The whole process is repeated until the given stopping criteria are met. The best chromosome in the final population expresses the solution.

STEP-BY-STEP PROCEDURE

Step 1: Choose a coding to represent problem parameters, a selection operator, a crossover, and a mutation operator. Choose the population size, m, crossover probability p_c, and mutation probability p_m. Initialize a random population of chromosomes $p^{(t)}$ of size m, and set $t = 1$.

Step 2: Evaluate each chromosome in population $p^{(t)}$ in terms of its fitness.

Step 3: Generate a new population, $p_n^{(t)}$, from the given population $p^{(t)}$ by some procedure of natural selection (reproduction operator).

Step 4: If stopping criteria are not met, go to step 5; otherwise, stop.

Step 5: Produce a population of new chromosomes, $p^{(t+1)}$, by operating on chromosomes in population $p_n^{(t)}$ (crossover and mutation operators).

Step 6: Replace population $p_n^{(t)}$ with population $p^{(t+1)}$ produced in step 5, increase t by one, and go to step 2.

DESCRIPTION OF THE IDENTIFICATION OPTIMIZATION PROBLEM

The identification technique is aimed at finding four elastic material properties of laminated composite structures through genetic algorithm by minimizing the difference between experimental and numerical eigen values. The numerical model is based on a finite element method for genetic algorithm and it is used to connect the prediction and experimental values through an updating process of natural frequencies followed by the determination of the frequency response. This is an inverse problem with the parameters in the mathematical model being adjusted repeatedly until the analytical responses match satisfactorily with those associated with the physical structure. During the iteration process, the genetic algorithm utilizes an error function to compare the theoretical ω and measured $\widehat{\omega}$ values as well as the initial and revised property estimates. The objective function F_r is defined as an error functional f, which depends on the eigen frequencies as follows:

$$f = |\widehat{\omega}_i - \omega_i| \qquad (i = 1, 2, 3, \ldots\ldots\ldots, n) \tag{1}$$

The optimization problem is formulated as the identification of the set of material parameters, which minimize the error functional:

$$\text{Min} f\,(E_x, E_y, G_{xy}, \nu_{xy}) \tag{2}$$

Since genetic algorithm requires a maximization objective function, then the above error function is transformed to a maximization objective function, which is expressed as:

$$F_r = 1 + |\omega_i - \widehat{\omega}|\,/\widehat{\omega}_i \qquad (i = 1, 2, 3, \ldots, n) \tag{3}$$

However, a computer program has been developed to implement the Genetic Algorithm for identification of material properties of laminated composite structures. Figure (1) illustrates the flow chart for integration of GA that can be adopted to numerical methods of analysis of design variables (E_x, E_y, G_{xy}, υ_{xy}).

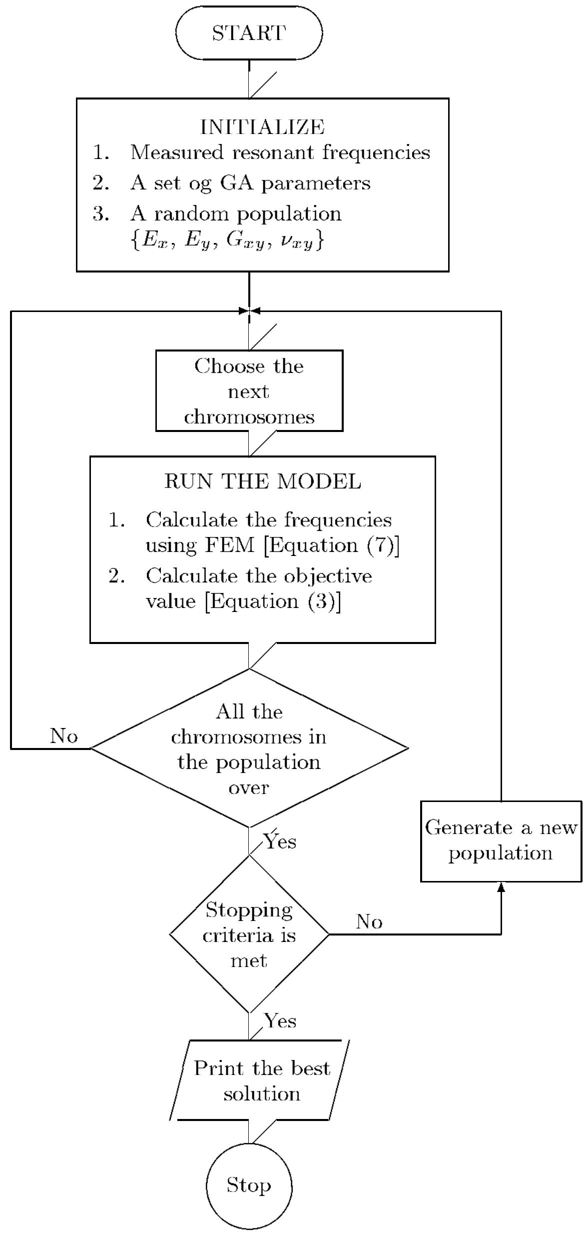

Figure 1. Integration of genetic algorithm

GA OPERATORS

In order to apply GA technique for solving the present problem, variables are first coded in a string structure. Binary coded strings having 1's and 0's are used in this technique. Reproduction is the first operation applied to a population. Reproduction selects good strings in a population and forms a mating pool. There exist a number of reproduction operators in GA literature, but the essential idea in all of them is that the above-average strings are picked from the current population and their multiple copies are inserted in the mating pool in a probabilistic manner. In the crossover operator, new strings are created by exchanging information among strings of the mating pool. Many different crossover operators exist in the GA literature. In most crossover operators, two strings are picked from the mating pool in a random base and some portions of the strings are exchanged between the strings. The need for the last operator, i.e., Mutation, is to create a point in the vicinity of the current point, thereby achieving a local search around the current solution. The Mutation is also used to maintain diversity in the population. Here a bit-wise mutation is performed bit by bit with a probability p_m.

ADVANTAGES OF USING GENETIC ALGORITHM

When applying traditional test methods, the determination of material properties of the complex materials, such as fiber reinforced composites, face serious difficulties. GA is a search method that helps to determine the parameters rather in a short time. GA works with a population of points instead of a single point. Because there are more than one string being processed simultaneously, it is very likely that the expected GA represents a global solution. Another advantage of a population-based search algorithm is that multiple solutions can be captured in the population easily. In trying to solve a multi-modal problem by GA with many local solutions, the search procedure may not easily get trapped in one of the local solution points. Since an initial random population is used, to start with, the search can proceed in any direction and no major decisions are made at the beginning. Later on, when the population begins to converge in some bit positions, the search direction narrows and a near-optimal solution is achieved.

THE FINITE ELEMENT MODEL

The present finite element formulation is based on the degenerated solid shell element [15]. In the finite element model, the laminated composite plate is discretized using six-nodded triangular elements with six degrees of freedom per node (three rotations and three transversal displacements). The element stiffness and mass matrices are derived using the principal of minimum potential energy. The element elastic stiffness matrix is given by:

$$[K]_e = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\det[J]d\xi d\eta d\zeta \tag{4}$$

Since each element may contain more than two plies with different ply orientation, the material properties in each element throughout the thickness may vary from layer to layer. Thus the elasticity matrix [D] may not be a constant matrix along the ζ axis. The variable ζ is suitably modified to ζ_k in any k^{th} layer such that ζ_k varies from -1 to $+1$ in that layer [16]. The change of variable is obtained from:

$$\zeta = -1 + \frac{1}{t}\left[-h_k(1-\zeta_k) + 2\sum_{j=1}^{k} h_j\right] \text{ and}$$

$$d\zeta = \left(\frac{h_k}{t}\right) d\zeta_k \tag{5}$$

where h_k is the thickness of k^{th} layer. The element elastic stiffness matrix then takes the form:

$$[K]_e = \sum_{k=1}^{n}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\det[J]\left(\frac{h_k}{t}\right)d\xi d\eta z d\zeta_k \tag{6}$$

where n is the total number of layers in the laminate and $[B]$ is the strain displacement matrix, and also ξ, η, ζ are local natural coordinates of an element. The element mass matrix is obtained from the integral:

$$[M]_e = \int_V [N]^T[\rho][N]\, dV \tag{7}$$

where $[N]$ and $[\rho]$ are the shape function and inertial matrices, respectively. The element elastic stiffness and mass matrices are evaluated first by expressing the integrals in the local natural coordinates ξ, η and ζ of the element and then performing the numerical integration by using Gaussian quadrature $(2 \times 2 \times 2)$. Then the element matrices are assembled to obtain the respective global matrices $[K]$ and $[M]$ after appropriate transformation. The free vibration analysis involves determination of natural frequencies (eigenvalues) from condition,

$$\left(-\omega_i^2 M + K\right)\phi_i = 0 \tag{8}$$

where ω_i is the i_{th} natural frequency and ϕ_i is its vibration mode. This is a generalized eigenvalue and is solved Byu the subspace iteration algorithm [17]. It is important to remark that the stiffness matrix depends on the linear elastic parameters that are to be identified, i.e.

$$K = K\left(E_x, E_y, G(xy, \upsilon_{xy}\right) \tag{9}$$

NUMERICAL RESULTS AND DISCUSSIONS

Estimate of Properties of Isotropic Plates

As a trial measure it was decided that these techniques be applied to a set of isotropic plates, one made of aluminum. Experimental results obtained by Sol [18] from tensile tests on specimens and from vibrational measurements are compared with those generated by the present algorithms. The geometric parameters and the experimentally determined frequencies are given in Table 1. The results of four trials using the genetic algorithm and experimental measurement of mechanical properties ($E\&\nu$) are presented in Table 2. It can be seen that the agreement of results is excellent. Another analysis was performed on a simply supported isotropic square plate using the above to identify the mechanical properties. The following data are taken from J. E. Jam [19]:

$$a = b = 0.3(\text{m}) \qquad h = 0.005(\text{m}) \qquad \rho = 1000(\text{kg/m}^3)$$

$$f_1 = 493.65(\text{Hz}) \quad f_{2\&3} = 1234(\text{Hz}) \quad f_4 = 1974.62(Hz)$$

where f_i are measured natural frequencies, a, b, h are plate dimensions and ρ is the plate unite mass. The estimated material properties of this plate is presented in Table 3.

Estimates For Mechanical Properties of Orthotropic Plates

Some laminated orthotropic composite plates whose material properties are to be identified are considered in this section. Each set of results identified is compared with those of other authors and with strain gauge measurements where available. The specimen with a carbon fiber reinforced epoxy plate made of uni-directional fiber plies with stacking sequence $(0°)_8$ was studied by Araujo et al. [12] and Frederiksen [20]. They identified the mechanical properties of this plate using the combined numerical, a higher order theory of finite element, and an experimental method using strain gauges and resonant frequency measurements. The plate under consideration had the

Table 1. Plate properties and experimental measurement of resonant frequencies of aluminium plate, B.C's (FF-FF)

Geometry & Properties			Mode No.	Freq.(Hz)
Length (m)	0.3985 ± 0.0001		1	119 ± 1
Width (m)	0.3500 ± 0.0001		2	165 ± 2
Mass (Kg)	1.9125 ± 0.0001		3	236 ± 2
Thickness (m)	0.00510 ± 0.00006		4	296 ± 4
Density (Kg/m^3)	2690 ± 30		5	326 ± 4

Table 2. Estimated elastic constants for an isotropic plate [Aluminium], using the GA

Material Property	Genetic Algorithm				Experimental Work
	Trial 1	Trial 2	Trial 3	Trial 4	
E (N/m^2)	7.10×10^{10}	7.11×10^{10}	7.10×10^{10}	7.10×10^{10}	$(7.00 \pm 0.05) \times 10^{10}$
ν	0.300	0.350	0.321	0.334	0.34 ± 0.01

dimensions given below and the study used the first five experimentally obtained natural frequencies (Hz) obtained experimentally:

$a = 0.24275\text{m}$ $\quad b = 0.1525\text{m}$ $\quad h = 0.00339\text{m}$

$f_1 = 161.23$ $\quad f_2 = 347.14$ $\quad f_3 = 476.93$

$f_4 = 495.69$ $\quad f_5 = 589.09\text{(Hz)}$

The comparison of the present results and some other investigators is presented in Table 4 for fore edges free B.C's. Identified results are in good agreement with those obtained by Araujo et al. [12] and Frederiksen [20]. (The percentage Error can be calculated as a $(f_{i,\text{Trial}} - f_{i,\text{Exp.}}/f_{i,\text{Exp.}}) \times 100$).

Estimates for Mechanical Properties of Anisotropic Plates

At this stage it is interesting to investigate the prediction of mechanical properties of laminated composite plates and show the performance and sensitivity of genetic algorithm techniques to evaluate these properties. This is the most important stage in the optimal analysis of laminated composite structures. The combination of experimental and numerical analysis, an adequate choice of objective function and constraints, together with the correct selection of design variables and optimization algorithms are the basic requirements for efficient optimal structural design. The specimen was for a carbon fiber reinforced epoxy plate made of uni-directional fiber plies with a stacking sequence

Table 3. Estimated elastic constants for an isotropic plate using GA & comparison with other results, B.C's (SS-SS)

Material Property	Genetic Algorithm				J.E. Jam [19]
	Trial 1	Trial 2	Trial 3	Trial 4	
E (N/m^2)	87.40×10^9	87.40×10^9	87.50×10^9	87.35×10^9	87.36×10^9
ν	0.30	0.30	0.30	0.30	0.30

Table 4. Estimated elastic constants for an orthotropic $(0^o)_8$ plate using GA & comparison with other results, B.C's (FF-FF)

Mechanical Property	Genetic Algorithm		Araujo [12]	Frederiksen [20]	Frederiksen [20]
	Trial 1	Trial 2	(Numerical)	(Analytical)	(Strain Gauge)
E_x (N/m^2)	106.75×10^9	107.0×10^9	107.8×10^9	107.1×10^9	113.2×10^9
E_y (N/m^2)	7.50×10^9	8.01×10^9	8.30×10^9	8.30×10^9	7.40×10^9
G_{xy} (N/m^2)	4.50×10^9	4.25×10^9	4.20×10^9	4.20×10^9	-
ν_{xy}	0.275	0.325	0.421	0.246	0.348
Error %	-1.9	0.780	0		

$(0, \pm 60_2)_s$ which was studied by Araujo et al. [12] and Frederiksen [20]. The plate dimensions and first five measured resonant frequencies for fore edges free B.C's are given below:

$a = 0.2227\text{m}$ $b = 0.1851\text{m}$ $h = 0.00126\text{m}$

$f_1 = 96.310$ $f_2 = 163.18$ $f_3 = 202.66$

$f_4 = 252.17$ $f_5 = 285.23(\text{Hz})$

The results for the three trials are compared with Araujo et al. [12] and Frederiksen [20] and are presented in Table 5, and all indicate good agreement.

CONCLUSION

In this investigation, a non-destructive optimization search technique for the identification of the four major (E_x, E_y, G_{xy}, ν_{xy}) mechanical properties of laminated composite plates based on the genetic algorithm has been established. The finite element method when associated with experimentally measured free vibration resonant frequencies enable the prediction of the mechanical properties of composite plate specimens within acceptable limits of accuracy through an error function. One program has been developed based on the finite element to show the capability and efficiency of genetic algorithm search techniques to identify the optimum results. Genetic algorithm is one optimization search technique which can be used with confidence for identification of mechanical properties of laminated composite materials.

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Table 5. Estimated elastic constants for a 10 layers $(0, +60_2)_s$ laminated composite plate using GA and comparison with other results, B.C's (FF-FF)

Mechanical Property	Genetic Algorithm			Frederiksen [20]	Araujo [12]
	Trial 1	Trial 2	Trial 3		
E_x (N/m^2)	118.00×10^9	117.75×10^9	117.75×10^9	118.50×10^9	118.80×10^9
E_y (N/m^2)	8.75×10^9	9.5×10^9	8.75×10^9	9.10×10^9	9.10×10^9
G_{xy} (N/m^2)	4.75×10^9	4.5×10^9	4.75×10^9	5.40×10^9	5.60×10^9
V_{xy}	0.275	0.300	0.325	0.304	0.318
Error %	0.34	0.304	0.0054	-	0

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