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A New Simple Analytical Model for Flow Analysis over Slender Delta Wings

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A simple analytical model is presented for determining the position and strength of the vortices over a slender delta wing at moderate to high angles of attack. The model is based on the inviscid incompressible flow theory. The slender wing assumption is a cornerstone of the current study on which the validity of the results depends. The lift force of the wing can be calculated using this simple analytical model. Comparison of the predicted lift coefficient results with the available experimental findings shows satisfactory agreement with slender wings, $AR \leq 1$. The method is currently applied with some revision to higher AR delta wings and the results will be presented in our future work.

NOMENCLATURE

AR	Wing aspect ratio
s	Wing semi span
V_∞	Free stream velocity
Vn	Normal Free stream velocity component, $V_\infty \sin\alpha$
W	Complex potential
Z	Complex physical plane
K	Similarity parameter
α	Angle of attack
ϵ	Wing half apex angle
ζ	Complex mapped plane
ξ	Real part of ζ
η	Imaginary part of ζ
φ	Velocity potential
Γ	Vortex strength

INTRODUCTION

Delta wings have been incorporated in many advanced and highly maneuverable aircrafts. The main advantage of this type of planform from design viewpoint is its low drag force in supersonic regime and good aerodynamic behavior at high angles of attack in the subsonic flow, a phenomenon which is vital for high performance airplanes.

Flow field over these wings at moderate to high angles of attack is different from that of conventional wings, mainly due to the presence of a pair of strong vortices. As a rough estimate, for a 70° swept delta wing, these vortices are formed over the wing surface at an angle of attack of about 10° and remain stable up to about 30° angle of attack. At higher angles, the vortices become unstable and vortex breakdown phenomenon occurs which deteriorates the performance characteristics of the aircraft.

The most important parameters for a thin flat delta wing in a symmetric configuration are aspect ratio and angle of attack. For developing the analytical model, some assumptions are to be made which confine the applicability of the model to low AR wings at low to moderate angles of attack. These assumptions are:

- Low AR, AR ≤ 1.
- Neglect viscous effect; i.e. do not consider separated shear layer.
- Stable vortices, do not consider breakdown phenomenon.

However, to author's knowledge there is no simple theory for predicting vortex characteristics in general and the existing ones can only address the phenomena to a certain angle of attack, except that of Polhamus [1]. Polhamus's theory predicts only lift coefficient and does not give any information about the character of the vortices and their effects on the pressure distribution over the wing surface. The goal of this research is, therefore, to develop a simple mathematical formulation to predict the vortex properties and their

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variations with angle of attack. Hence, one can use this result to obtain surface pressure distribution and consequently, lift, pitching moment and induced drag coefficients.

ANALYTICAL APPROACH

Consider the flow past a thin flat delta wing with half-apex angle å at an angle of attack a. The coordinates used in this investigation are with respect to the body axis system as shown in Figure 1. Note that xy plane is the plane of symmetry. At moderate angles of attack, the separated leading edge flow produces two distinct vortices. For the sake of simplicity, it is assumed that these vortices are straight filaments with variable strengths along their axes, which is a good assumption for slender delta wings as suggested in references [2-12]. However, at high angles of attack, these assumptions fail due to the vortex bursting phenomenon and secondary vortices. The governing equation for inviscid incompressible flow field is the Laplace equation of the form:

$$\nabla^2 \phi = 0 \tag{1}$$

For a slender delta wing, having a small aspect ratio, variations of the flow properties along z direction are much smaller than those of the other two directions [13-15]. Thus, Eq.(1) reduces to:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{2}$$

This equation shows that the problem can be solved at each cross section of the wing as a 2D problem. Using this method, the resultant aerodynamic lift, induced drag and pitching moment coefficients can be calculated.

COMPLEX POTENTIAL

It is desirable to solve the problem, Eq. (2) in an arbitrary cross section of the wing. Since the problem has been reduced to a 2D case, it can be solved using complex variable techniques. The Z plane is considered perpendicular to the wing centerline at an arbitrary station as shown in Figure 2. The wing plane is rotated such that the vertical x axis becomes horizontal. The free stream velocity normal to the wing plane is $V_n = V_\infty \sin(\alpha)$. The complex velocity potential can be obtained by transforming the physical plane using the following mapping:

$$Z = \frac{1}{2}\left[\zeta - \frac{s^2}{\zeta}\right] \tag{3}$$

The geometry of the mapped plane is the flow over a circular cylinder as shown in Figure 3. The following transformation is proposed for the solution:

$$W = \frac{1}{2}\left[\zeta + \frac{s^2}{\zeta}\right] \tag{4}$$

The result of the above transformation is shown in Figure 4. As seen, the transformed wing is a streamline. Therefore, there is no need to solve the Laplace equation. By reverse transformation, it can be shown

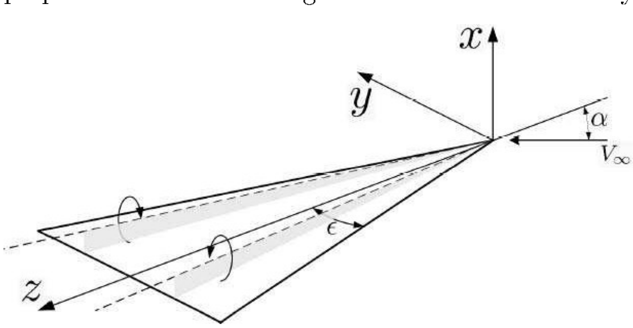


Figure 1. Geometry of coordinate system and free stream velocity

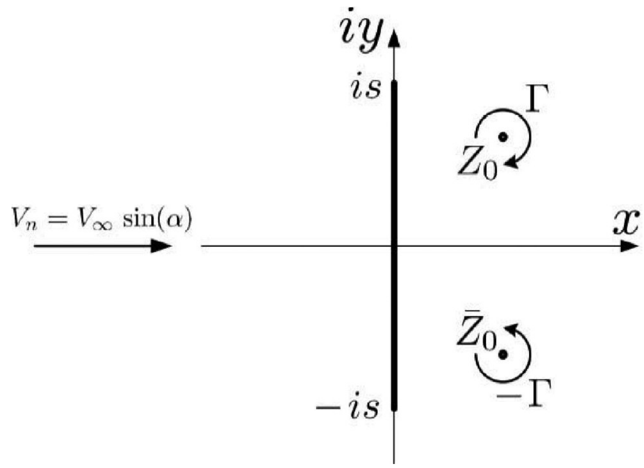


Figure 2. Wing cross section and normal velocity component past the wing

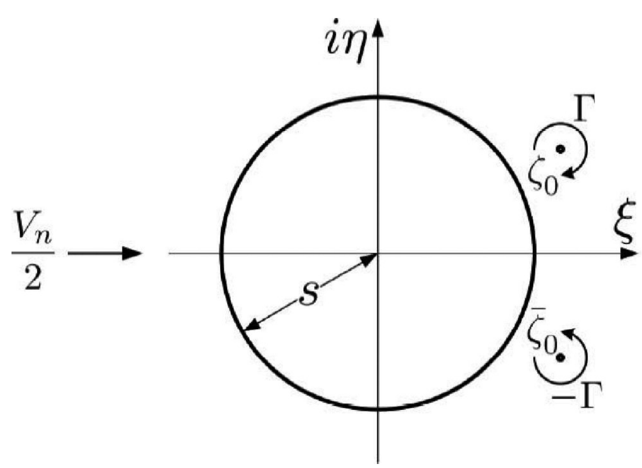


Figure 3. Wing cross section after first conformal mapping

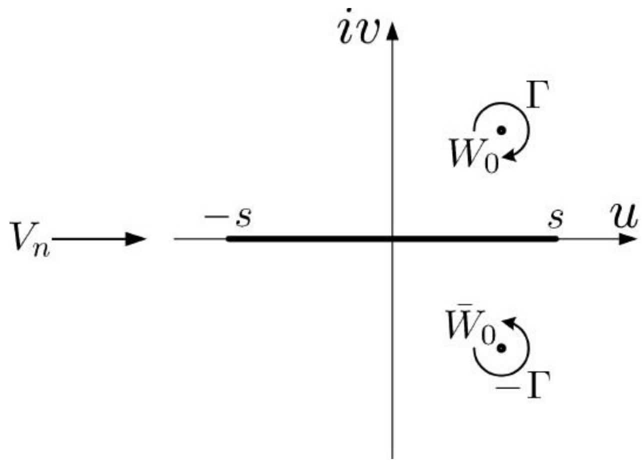


Figure 4. Wing cross section after second conformal mapping

that the velocity potential in the ζ plane is:

$$W(\zeta) = \frac{V_n}{2}\left(\zeta + \frac{s^2}{\zeta}\right) + \frac{i\Gamma}{2\pi}\left[\log\frac{\zeta-\zeta_0}{\zeta-\widehat{\zeta}} + \log\frac{\zeta-(s^2/\zeta_0)}{\zeta-(s^2/\widehat{\zeta})}\right] \tag{5}$$

Where G is the vortex strength and ζ_0 and $\bar{\zeta_0}$ are positions of the transformed vortices in the ζ plane. From the ideal flow theory, the velocity at a sharp edge has to vanish. This is known as Kutta condition. Hence, the velocity at $\zeta = \pm$is is zero. Applying this condition on Eq. (5) results in:

$$\frac{\Gamma}{\pi s V_n} = \frac{\left(s^2 + |\zeta_0|^2\right)^2 - 4s^2\eta_0^2}{2\eta_0\left(|\zeta_0|^2 - s^2\right)s} \tag{6}$$

where η_0 is the imaginary part of ζ_0 in the ζ plane. Further, the induced velocity on the vortices should be zero too. This condition is to make sure that the vortices will remain stationary over the wing surface. The complex velocity of the free vortex at ζ_0 is:

$$u_0 - i\nu_0 = \lim_{Z\to Z_0}\frac{d}{dZ}\left[W(\zeta) - \frac{i\Gamma}{2\pi}\log(Z - Z_0)\right] \tag{7}$$

This relation takes the contribution of the wing and the free vortex at $\bar{\zeta_0}$ into account. Equation (7) holds for a pure 2D problem. However, the contribution of the chord wise free stream velocity, V_∞, on the vortex should be considered. In fact, axial velocity has a component normal to the vortex line because vortices are not parallel to the wing plane (see Figure 1). Thus, the induced velocity component of V_∞ on the vortex is important. It can be shown that if this velocity is not considered, the problem becomes ill-conditioned. The induced velocity on the vortex is of the form:

$$(u_0 - i\nu_0)_{ind} = -\frac{V_n}{sK}\widehat{Z} \tag{8}$$

Where K is a similarity parameter defined by:

$$K = \frac{\tan\alpha}{\tan\epsilon} \tag{9}$$

Note that in these equations, $V_\infty = V_n/K$. Subsituting Eq. (7) into Eq. (6) and after some mathematical manipulations, the following relation is obtained,

$$u_0 - i\nu_0 = \lim_{\zeta\to\zeta_0}\frac{d}{d\zeta}\left[W(\zeta) - \frac{i\Gamma}{2\pi}\log(\zeta-\zeta_0)\right]\left(\frac{d\zeta}{dZ}\right)_{Z=Z_0} - \frac{i\Gamma}{4\pi}\left[\frac{d^2Z/d\zeta^2}{(dZ/d\zeta)^2}\right] - \frac{V_n\widehat{Z}}{}K_s \tag{10}$$

The latter is the complex velocity potential of the flow field at the vortex core in the Z plane. The additional derivatives are due to the transformation from ζ plane to the Z plane. The boundary condition for a vortex to be stationary is:

$$u_0 - i\nu_0 = 0 \tag{11}$$

This condition provides two algebraic equations, which with Eq. (6) form a nonlinear system. This system can be solved to obtain ζ_0 and G.

LIFT ESTIMATION

Since the strength and position of the vortices are known, the pressure distribution around the wing can easily be obtained. The computational treatment can be carried out using the Blasius theorem [11, 13, 14]. This theorem states that the net force per unit length of a body is,

$$F_x - iF_y = \frac{i\rho}{2}\oint_C\left(\frac{dW}{dZ}\right)^2 dZ \tag{12}$$

Where c is the boundary of the body and W is the velocity potential. If the velocity potential function is known, this integration is straightforward via any numerical method.

RESULT

The nonlinear systems of Eqs. (6) and (11) are solved numerically to obtain the nondimensional circulation strength and vortex position in the ζ plane. Then, using Eq. (3), one can determine the vortex position in the physical plane. The only required parameter is the similarity parameter, K, and the results can be presented in a 2D graph as shown in Figs. 5 through 7. In these figures, the results are shown for various values of K.

In Figures 5 and 6 both the nondimensional spanwise (x/s) and normal (y/s) position of the vortex versus K are shown. The results are also compared to the experimental results of ref.[6]. These figures

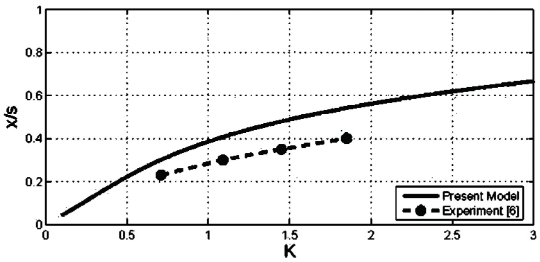


Figure 5. Nondimensional normal vortex position above the wing; solid line is the result of the present model and dashed line is from experiment [6].

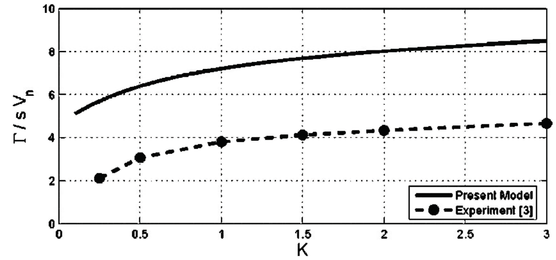
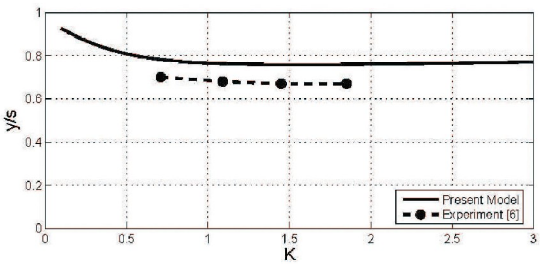


Figure 7. Nondimensional vortex strength above the wing; solid line is the result of the present model and dashed line is from experiment [3].



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clearly show that the present simple analytical method predicts these variations fairly well and the comparisons are satifactory. It should be noted that the experimental results for different sources are slightly different too. Although the results show about 10 percent difference with the corresponding experimental data, the trend is qualitatively correct.

As seen from these figures, Figs 5 and 6, increasing K results in an upward movement of the vortices at almost the same span wise position. This observation is in agreement with the experimental results of Ref.[5, 6]. For values of K less than 1.89, the vortices shift outward the wing surface. The most inner position of the vortices occurs for K =1.89 where they are located at about 75.8% of the local wing semi span. There is not any experimental data to validate the above phenomenon; however results of Reference [3] and [6] reveals that the span-wise position of the vortices is more or less constant.

The effects of K on the vortex strength is shown in Figure 7. The dashed line represents experimental results and solid line is the results of present work. Although differences in the results are noticeable, the ratio is almost constant. Thus, the calculated vortex strength is proportional to the experimental value. This figure clearly shows that this simple method predicts the trend of the effect of K on the vortex strength very well. From this figure it is seen that the experimental data for all values of K are lower than the predicted one by an amount of about 1/1.6, that is, if the analytical results are multiplied by this value, 1/1.6, the differences would be minimized. Further more, it is shown that the variations of vortex strength with K follows a power curve, ref. 12, and as seen from Figure 7, our simple analytical method supports this finding too.

The differences between the results of the present model and the experimental data seen in Figure 7 are likely to be due to the physics of the vortices. In practice, the vortices over delta wing surface are not simple filaments. Instead, they are shear layers originating from the leading edge and rolling over the wing surface. Therefore, there is a vorticity distribution in the field. The difference between the calculated and experimental vortex strength might be due to the replacement of this vorticity distribution with a single vortex. Obviously, such a vortex filament should be stronger than the core vortex seen in real experiments. As mentioned before, the lift force and its relation with angle of attack can be calculated using this simple method. The results for 75 and 80 degrees sweep delta wings at different angles of attack are shown and compared to the experimental data in Figs. 8 and 9, respectively. Note that the present method is valid only where the vortices are formed completely over the wing surface, thus, at low angles of attack where the flow over the wing is completely potential, the method is not very accurate. Moreover, at high angles of attack where the vortices over the wing burst, the model fails too. This is expected since the model is developed for potential flow, hence it is not able to predict stall. As seen from figures 8 and 9, the accuracy of the calculated lift coefficient is much better than that for the vortex position and strength, Figs. 5-7. The reason is the method of calculation of lift coefficient. In contrast to the vortex position and strength, lift coefficient is an integral quantity based on the pressure distribution over the surface. Therefore, the different sources of error may cancel each other out. Further studies are underway to fulfill this defect, hence enabling us to predict the variations of lift, and pitching moment with angle of attack for delta wings with different leading-edge sweep angles operating at low to moderate angles of attack prior to vortex breakdown.

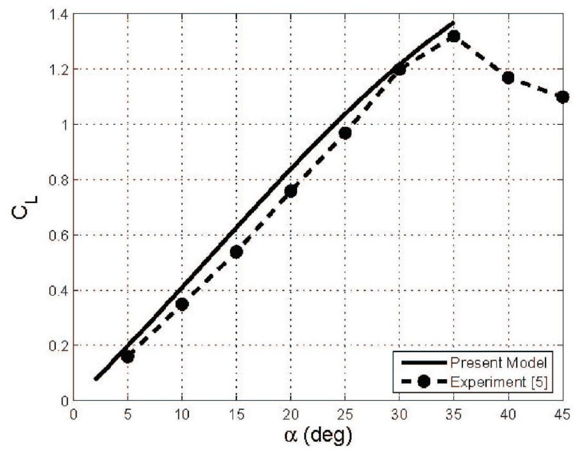

Figure 8. Lift coefficient for a 75° delta wing; solid line is the result of the present model and dashed line is from experiment.

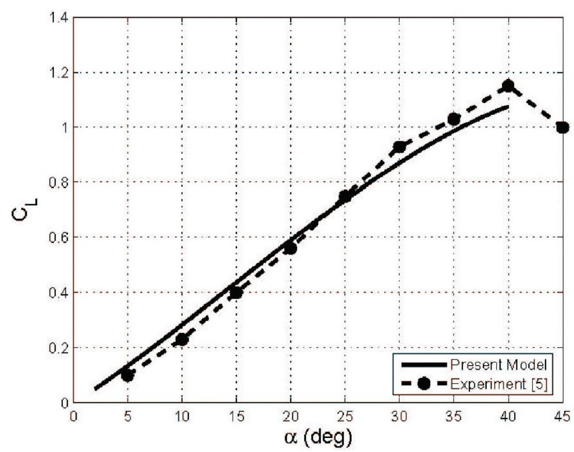

Figure 9. Lift coefficient for a 80° delta wing; solid line is the result of the present model and dashed line is from experiment.

CONCLUSION

A simple analytical model for the calculation of vortex characteristics over a slender delta wing is proposed. The limitations of the method are pointed out and the results are compared to the experimental data. Although the method is very simple, the results are promising. The results indicate that increasing K leads to stronger and larger vortices over the wing, which is confirmed by the existing experimental data. The agreement between the obtained lift coefficient within the limitations of the method is satisfactory. Further work is underway to improve both the accuracy and ranges of validity of the present method. However, comparing this very simple method to other ones in the literature, it is clear that this scheme has the following advantages:

- It is very simple and is completely based on the physics of the flow
- It is able to predict both vortex strength and positions over the wing surface

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