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Orbital Perturbation Effect on LEO Satellite Trajectory

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A satellite, in its orbit, is affected by perturbing forces, such as atmospheric drag, solar radiation pressure, Earth oblateness, and gravity of the celestial masses (other than the Earth). In this paper, the effects of these forces on the trajectories of different types of LEO satellites and a sample satellite (Z-SAT) are discussed. A few analyses are done on these orbits and the relevant results are categorized as substantial logics of perturbation influences on Keplerian parameters of the orbits. The "Variation of Parameters" Method is used for analyses.

INTRODUCTION

Applying two-body motion theorem to a satellite results in ideal path called ideal orbit. In the ideal orbit case, the six Keplerian parameters are just related to Earth's gravity field. Beside this dominant source, there are so other perturbation sources that vary the orbital parameters. Earth oblateness (non- homogeneity), atmospheric drag, solar radiation pressure, gravity' of celestial masses, magnetic field of the Earth, solar wind, and magnetic turbulence are some of these sources [1]. The orbit and attitude of a satellite are both affected by perturbation. In order to prevent their impacts, the satellite navigation system designer should predetermine their approximate effect, theoretically or experimentally [1,2].

In the past decade, many papers were presented on the estimation of satellite trajectory, using data obtained from magnetometers, sun sensors and star sensors [3,4,5,6]. Various analyses have been done on orbital perturbation. For low-Earth-orbiting satellites where atmospheric drag is the dominant perturbation source, several stability analyses have been performed on gravity-gradient stabilized ones [7,8]. A complete investigation on orbital perturbation of the geostationary orbit has been done in the literature [9].

In this paper, a comprehensive investigation on the effect of orbital perturbation on trajectory of low- Earth-orbiting satellites is carried out. The perturbation sources considered in the paper are Earth oblateness, atmospheric drag, and solar radiation pressure. Variation of Parameters method is used in the analyses [10,11]. Several circular and elliptic orbits are analyzed and results categorized as the general principels governing the orbital perturbation effect on low-Earth-orbiting satellites.

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PERTURBATION RESOURCES

Various perturbation sources can affect the trajectory of the LEO (low-Earth-orbiting) satellites (hereafter called satellite for simplicity) and hence deviate it from its ideal one. The satellite motion equation in the ideal orbit is as follows:

$$\frac{d^2\bar{r}}{dt^2} = -\mu\frac{\bar{r}}{r^3} \tag{1}$$

where:

- $\bar{r}$, is the satellite displacement vector with its origin at the Earth center.
- μ, is a physical parameter dependent on mass of the Earth and the universal gravitational constant.
- t, is time.

In the actual case, the acceleration caused by perturbation forces, $\vec{\gamma}_p$ is added to the right hand side of Eq. (1). Then we have:

$$\frac{d^2\bar{r}}{dt^2} = -\mu\frac{\bar{r}}{r^3} + \vec{\gamma}_p \tag{2}$$

Determining this acceleration and integrating the above equation, the actual path of satellite could be found. Since knowledge of Keplerian parameters is very important in attitude and orbital control of a satellite, "Variation of Parameters" method is used to determine how much the orbital perturbation can affect the ideal orbit. This method gives us the variation of each parameter, separately and independently [12].

The perturbation sources to be considered in this paper are Earth oblateness, atmospheric drag, solar radiation pressure, and gravity of celestial masses (other than the Earth).Among these, the effect of celestial masses is negligible for low-Earth-orbits.The potential function of gravity of a celestial mass is:

$$U_{PJ} = \frac{\mu_{pj}}{r_{pj}} \left[1 - \frac{1}{2}\left(\frac{r}{r_{pj}}\right)^2 + \frac{3}{2}\left(\frac{r}{r_{pj}}\right)^2 \cos\psi_j \right] \tag{3}$$

As Figure 1 shows, the above function is related to j'th celestial mass. r , r_{pj} are distances of the satellite mass center and celestial mass center from the Earth mass center, respectively. ψ_j is the angle between these two position vectors. Even the nearest celestial masses, like the Moon, have no effective influence on the trajectory of the satellite when compared to the effect of other sources. For example, for the Sun and the Moon, the ratios of the two distances mentioned above are:

$$\left(\frac{r}{r_{pj}}\right)^2_{moon} = 3.384 \times 10^{-4}$$

$$\left(\frac{r}{r_{pj}}\right)^2_{sun} = 2.234 \times 10^{-9} \tag{4}$$

These values indicate that potential functions of the Moon and the Sun have nearly a fixed value in different locations of the satellite. Due to the fact that variation of the Keplerian parameters is related to variation of the potential functions, this orbital source is negligible.

Vector a is defined as the Keplerian vector [10]. Entries of this vector are:

a, Semi-major axis

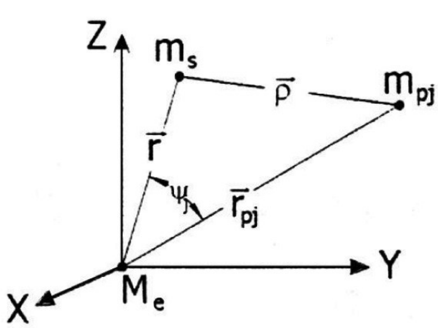

Figure 1. Geometrical parameters involved in determining the effect of gravity of a celestial mass

e, Eccentricity

i, Inclination angle

Ω, Right ascension of the ascending node

ω, Argument of pericentron

M, Mean anomaly

Earth oblateness effect appears in the last three parameters of the Keplerian vector as follow [10]:

$$\frac{d\Omega}{dt} = -\frac{3}{2}\frac{nJ_2\cos i}{2\left(1-e^2\right)^2}\left(\frac{R_e}{\alpha a}\right)^2 \tag{5a}$$

$$\frac{d\omega}{dt} = \frac{3}{4}\frac{n\,J_2\left[1-5\cos^2 i\right]}{\left(1-e^2\right)^2}\left(\frac{R_e}{a}\right) \tag{5b}$$

$$\frac{dM}{dt} = n + \frac{3}{4}\frac{n\,J_2\left[3\cos^2 i-1\right]}{\left(1-e^2\right)^{3/2}}\left(\frac{R_e}{a}\right)^2 \tag{5c}$$

where,

$$\text{Mean Motion}: n = \sqrt{\frac{\mu}{a^3}}$$

Mean Radius of the Earth :$R_e = 6371\,Km$

$$J_2 = 1082.6 \times 10^{-6} \tag{6}$$

Atmospheric drag could result in decreasing the satellite orbiting height by the following equation:

$$\frac{da}{dt} = -\rho\sqrt{a\mu}\,\frac{C_D S_D}{m_s} \tag{7}$$

where

ρ, is air's density (assumed to be related only to the height)

C_D, is the drag coefficient

S_D, is satellite's area normal to velocity direction

m_s, is satellite's mass.

The perturbation force caused by the solar radiation pressure is computed from the following relation,

$$F_r = \frac{1358}{1.0004 + 0.0334\cos D}\,\frac{S_R C_p}{m_s} \tag{8}$$

in which,

D, is a coefficient assumed to be constant, because its changes are not considerable

S_R, is satellite's area normal to solar radiation direction

C_p, is radiation factor (between 1 and 2).

The solar radiation pressure will affect all Keplerian parameters, especially a, e, i. According to the "Variation of Parameters" method, the following relations are integrated [10]:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \{e \sin \theta + [1 + e \cos \theta] S\} \quad (9a)$$

$$\frac{da}{dt} = \frac{\sqrt{1-e^2}}{na} \{\sin \theta R + [\cos \psi + \cos\theta] S\} \quad (9b)$$

$$\frac{di}{dt} = \frac{1}{na\sqrt{1-e^2}} \frac{r}{a} \cos \{\theta + \omega\} W , \quad (9c)$$

where R, S, W are components of the perturbation acceleration vector along the $\hat{R}$, $\hat{S}$, $\hat{W}$ directions of the body coordinate system, respectively. Figure (2) shows this frame.

Z-SAT

Schematic diagram of this micro satellite is presented in Figure (3). Z-SAT is an axi-symmetric satellite and its 700-km circular orbit is sunsychronous [13].

EARTH OBLATENESS

In the case of low-Earth-orbit satellites, the orbit is chosen sunsynchronous by selecting the proper inclination angle. Using Eq. (5,a), the inclination angle

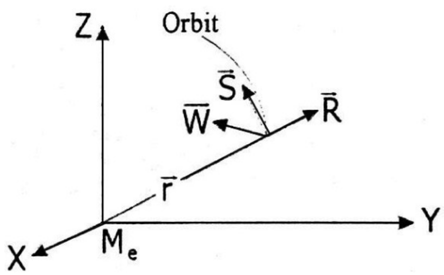

Figure 2. The body coordinate system

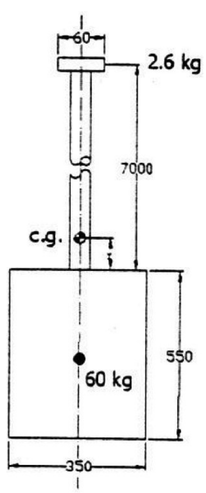

Figure 3. General physical and geometrical specifications of Z-SAT

is specified so that the variation rate of Ω would be equal to the spin rate of the solar frame. In this paper, the sunsychronous orbits are analyzed. Increasing the satellite orbiting height (up to 1000 km) decreases air's density and consequently drag effect. For example, height decrease of a 400-km circular orbit is nearly 95 times that of the corresponding decrease of a 700-km circular orbit. A computer program is written for analyzing the perturbation effect. Using this program, height variation due to the drag was determined. Figure (4) shows results of some of these analyses. 4,a and 4,b present the decrease of satellite orbiting height for circular orbits during one year time, while 4,c and 4,d show the corresponding results for elliptic orbits. Drag effect is so severe for orbits lower than 280 km that the satellite incides surface of the Earth after only a few working days if there is no compensation. In the present analyses the relevant parameters were substituted by the constants values listed below:

$$C_D = 1.05 \, , \, m_s = 150 \, \text{kg} \, , \, S_D = 0.8 \, \text{m}^2 \; . \quad (10)$$

Based on the analyses done on various orbits, polynomials are given to estimate the decrease of satellite orbiting height in different orbiting ranges. Table (1) shows these polynomials for circular and elliptic orbits.

Figure (5) shows altitude decrease of Z-SAT over 5 years time (operational life of this satellite), caused by the atmospheric drag. The results show that the drag has negligible effect on trajectory of this micro satellite.

References [3,5,8] also present atmospheric drag effect on trajectory. However, the results provided here are general and not for a specific height [3] or orbit [5,8].

SOLAR RADIATION PRESSURE

Contrary to the atmospheric drag, the solar radiation pressure depends on several parameters, such as orbital height, satellite surface area, and direction of the radiation with respect to orientation of the satellite. In each iteration, we should calculate the satellite area normal to the radiation. To do this, four coordinate systems are defined. Figure (6) shows these systems. Frame RSW is the body coordinate system, Frame xyz is the orbital frame that moves along the orbit. XYZ is the Earth inertial frame with its origin at the Earth center. And finally frame $X_0 \, Y_0 \, Z_0$ is the solar frame with its origin at center of the Sun and rotating around Z_0.

Defining point A as the satellite's center of mass,

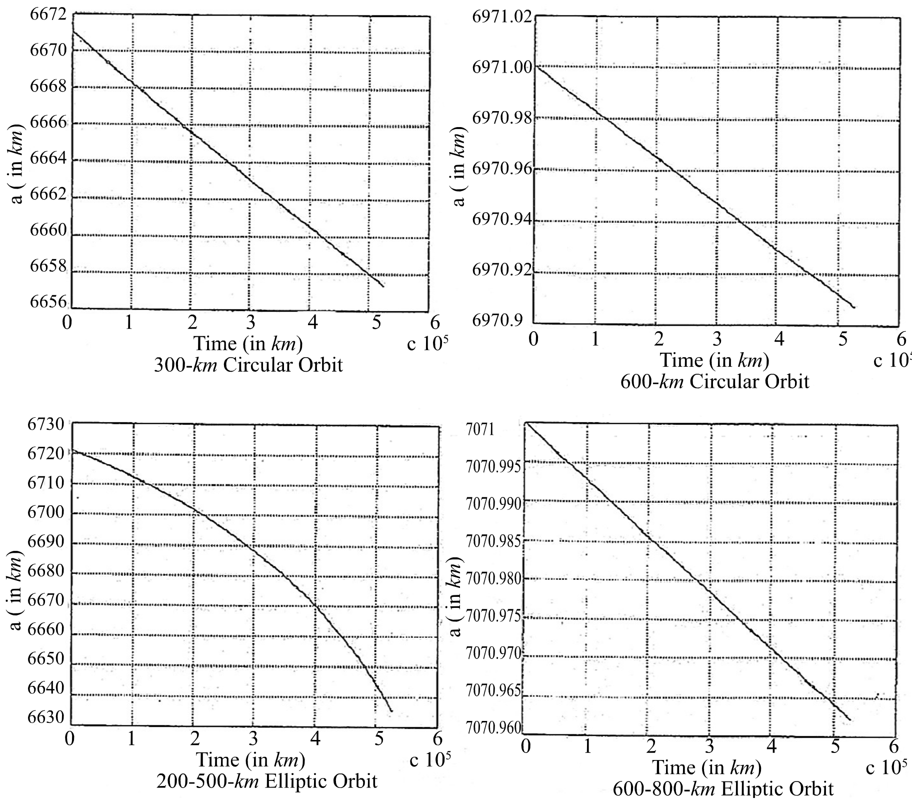

Figure 4. Decrease of satellite orbital height due to atmospheric drag

Table 1. Height decrease estimator polynomials

Circular Orbit	
Ranges of height	*Polynomial*
280-300	$-0.4 + 134$
300-400	$0.009\,a^2 - 0.719\,a + 152.3$
400-500	$0.0002\,a^2 - 0.1535a - 39.7$
500-600	$3.5 \times 10^{-5}\,a^2 - 0.0421a + 12.75$
600-800	$2.5 \times 10^{-6}a^2 - 0.039\,a + 1.52$

Elliptic Orbit		
Minor height	*Range of* major height	*Polynomial*
300	400-1000	$-133 \times 10^{-7}a^3 + 0.0003a^2 - 0.2202a + 58.87$
400	600-1000	$137.5 \times 10^{-6}a^2 - 0.003025a + 2.07$
600	800-1000	$-6 \times 10^{-5}a + 0.085$
800	900-1000	$-1.5 \times 10^{-5}a + 0.019$

its location vector in the solar frame is.

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = T_y\,(\omega_a t)\; t_x\,(23.5^\circ)\; T_z\,(-\Omega)\; T_x\,(-i)$$

$$T_z \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} 149.6 \times 10^6 \\ 0 \\ 0 \end{Bmatrix} \text{(km)} \quad (11)$$

in which, ω_0, is rotation rate of the solar frame.

$$\omega_0 \frac{2\pi}{365 \times 24 \times 3600}(rad/s)\ . \quad (12)$$

T is the rotation tensor and i is the inclination angle.

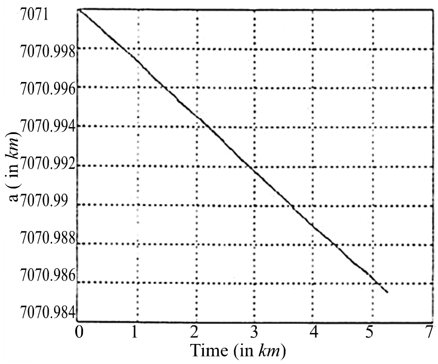

Figure 5. Decrease of Z-SAT orbital height due to atmospheric drag

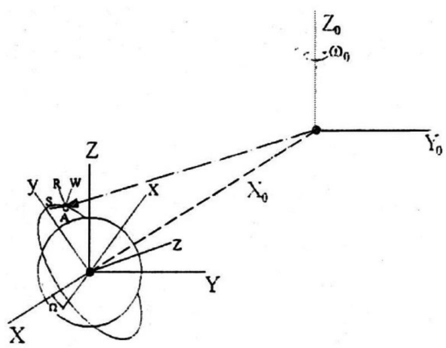

Figure 6. Coordinate systems used for computing the direction of radiation

The components "x" and "y" are:

$$x = a\left(\frac{e + \cos\theta}{1 + e\cos\theta} - e\right)$$

$$y = a\frac{\sin\theta\,(1 - e^2)}{1 + e\cos\theta} \qquad (13)$$

In order to determine the direction of radiation, vector $\vec{O}_0 A$ is calculated in the body coordinate system. That is:

$$\vec{O}_0 A = T_z(\omega + \theta)\, T_x(\Omega)\, T_x(-23.5^\circ)\, T_y(-\omega_0 t) \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} \qquad (14)$$

In this relation, θ is the true anomaly measured from the pericentron. According to Eq. (12), the satellite is in eclipse when any of the following conditions is satisfied:

$(i)\ O_0 A(1) \leqslant 149.6 \times 10^6$

$(ii)\ O_0 A(2) < -6371\ or\ O_0 A(2) > 6371$

$(iii)\ O_0 A(3) < -6371\ or\ O_0 A(3) > 6371\ .$ (15)

Thus, the satellite effective area for radiation could be found from the following relation:

$$S_r = |R_{com}|\, S_R + |S_{com}|\, S_S + |W_{com}|\, S_W\ . \qquad (16)$$

The parameters R_{com}, S_{com}, W_{com} are components of the unit vector of $\vec{O}_a A$ along the three directions R,S,W of the body coordinate system respectively. While, the parameters S_R, S_S, S_W are projections of side areas of the satellite on these three directions respectively.

The S_S, S_S and S_W, calculated for Z-SAT, are as follows:

$$S_R = S_S = S_W = 0.8\,(\mathrm{m}^2)\ , \qquad (17)$$

while C_P (radiation factor) and m_s (satellite's mass) for this satellite are found to be:

$$C_P = 1.5, \quad m_s = 150\,(\mathrm{kg})\ . \qquad (18)$$

The above-mentioned process was preceded for various cases, and the solar radiation pressure effect on many orbits was determined. Figures (7) and (8) show the variation of Keplerian parameters over a one-year period, due to the solar radiation pressure for a 200-km circular and a 600-1200 km elliptic orbit, respectively.

In contrast to the drag perturbation, which is not dependent on initial value of Ω, the solar radiation pressure is related to Ω and also to the initial value of ω. Hence, each Figure of this section has four subfigures inside itself. The procedure was taken for other circular orbits of different heights, such as 200, 400, 700, 1000, and 1500 km. The procedure was also applied to different elliptic orbits, such as 200-400, 200-800, 400-700, and 600-1000 km.

The results of the analyses on an elliptic orbit are presented in Figure (8). Minor and Major radiuses of the orbit are 600 and 1200 km. The following principels are revealed from the results:

1. Circular Orbit
   (a) The inclination angle has a "Cosine" shape when the initial angle is $0 = 0{,}90^\circ$, while the profile takes a "Sine" shape when $\Omega = 45^\circ, 135^\circ$. This means that $D.C. = A_t \cos(\pi - 2\Omega_0 - \omega_i t)$, where,

      $D.C.$ is deviation curve

      A_t and ω_i are two factors dependent on the orbital height.

      A_i decreases with an increase in the orbital height.

   (b) Eccentricity has a very small variation. Mean value of the eccentricity increases up to 6×10^{-6} in one year.
   (c) Mean value of the height could increase or decrease depending on initial value of Ω. The inclination sign could be determined by the sign of "$\sin 4\Omega_0 - \cos 4\Omega_0$". However, the higher the orbital height, the higher the rate of inclination.
2. Elliptic Orbit

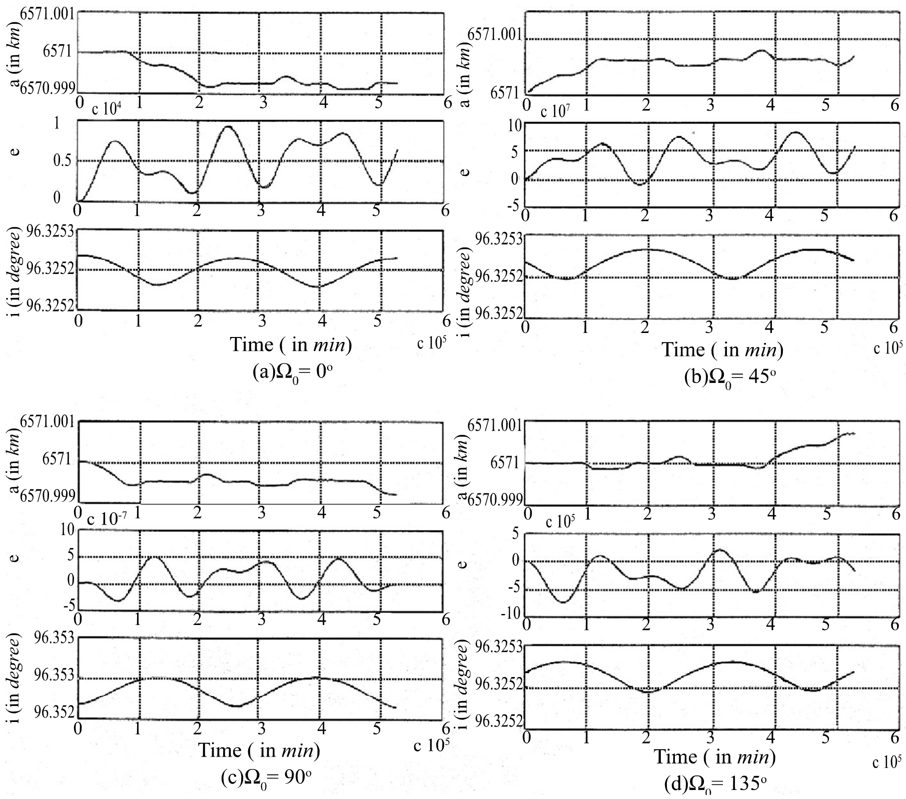

Figure 7. Variation of (Keplerian parameters due to solar radiation pressure for a 200-km circular orbit

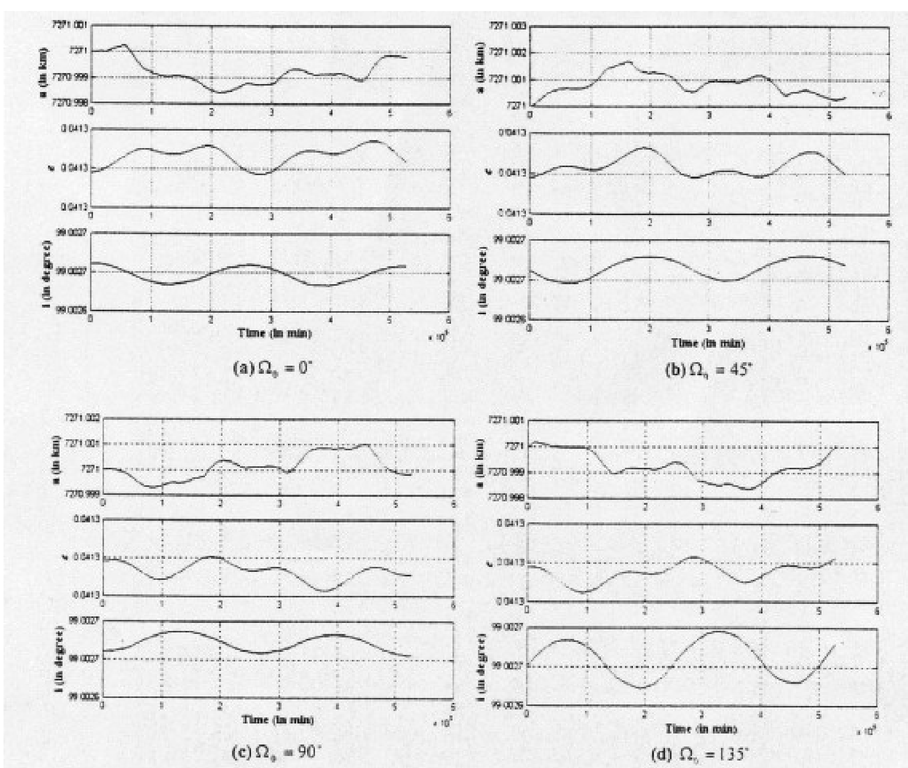

Figure 8. Variation of Keplerian parameters due to solar radiation pressure, for a 600-1200 km elliptic orbit

Fundamentally, in this case, both magnitude and profile of deviation of Keplerian parameters have no considerable dependency on initial value of ω.

The elliptic orbits have height variation in its nature, so the governing principals for these orbits could be derived from the circular orbits, with some modifications that have to be done.

(a) The variation profile of the inclination angle for the elliptic orbits is the same as circular orbits.
(b) Eccentricity is nearly fixed.
(c) Height is nearly fixed.

CONCLUSION

In this paper, the effect of main perturbation sources, such as Earth oblateness, atmospheric drag, and solar radiation pressure on the trajectory of low-Earth-orbiting satellites were investigated. The effect of gravity of celestial masses other than the Earth was ignored as it was deemed too small compared to other sources.

Earth oblateness effect was considered under the assumption that the orbits were sunsynchronous. It was supposed that air density is only a function of height. Based on this assumption, drag effect on various circular and elliptic orbits was computed for a duration of one year. The relevant results were categorized and polynomials were proposed for estimation of satellite orbiting height decrease during a one-year period. In orbits more than 400-km height, since the drag effect is linearly proportional to the time, the height decrease calculated by these polynomials over one year time could simply be multiplied by the number of years so that we could find the height decrease over several years. Nevertheless, for the orbits less than 400-km height, this must be done year by year. Furthermore, the drag effect decreases drastically with height increase.

In order to calculate the direction of solar radiation, the satellite's location vector was transformed from the solar frame to the body frame. Solar flux was assumed constant. The effect of solar radiation pressure was investigated on several circular and elliptic orbits with different initial values of right ascension of the ascending node and argument of perigee. Solar radiation pressure effect was shown to be so small in low-Earth-orbits that it could be neglected. However, its profile depends on height, Ω, and ω.

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